MATH 150 Review 1 Key Exam 2

1. Find the following limits:

   (a) $\lim_{t\to 2} \frac{t^2-4}{t^3-8}$

   factor and cancel $= \dfrac{1}{3}$

   (b) $\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}$

   rationalize $= 0$

   (c) $\lim_{x\to 1} e^{x^3-x}$

   $= 1$

   (d) $\lim_{h\to 0} \frac{(h-1)^3+1}{h}$

   $= 3$

   (e) $\lim_{r\to 9} \frac{\sqrt{r}}{(r-9)^4}$

   if r approaches from the left or right the denominator is small and positive $= \infty$

   (f) $\lim_{s\to 16} \frac{4-\sqrt{s}}{s-16}$

   rationalize or factor $= -\dfrac{1}{8}$

   (g) $\lim_{x\to 8^-} \frac{|x-8|}{x-8}$

   $= -1$

2. Use the definition of the derivative to find $f'(2)$ if $f(x) = x^3 - 2x$

   $$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \ \ldots \ = 10$$

3. Prove, using the δ , ϵ definition of a limit, that $\lim_{x\to 5} (7x-27) = 8$

   $$\delta \le \frac{\epsilon}{7}$$

4. Find an equation of the tangent line to $y = 9 - 2x^2$ at the point $(2, 1)$.

   $$y - 1 = -8(x-2)$$

5. Let:

$$f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 3 - x, & 0 \le x < 3 \\ (x-3)^2, & x > 3 \end{cases}$$

(a) Evaluate $\lim_{x\to 0^+} f(x)$ $= 3$

(b) Evaluate $\lim_{x\to 0^-} f(x)$ $= 0$

(c) Evaluate $\lim_{x\to 0} f(x)$ $=$ DNE

(d) Evaluate $\lim_{x\to 3^+} f(x)$ $= 0$

(e) Evaluate $\lim_{x\to 3^-} f(x)$ $= 0$

(f) Evaluate $\lim_{x\to 3} f(x)$ $= 0$

6. The displacement of an object is given by $s(t) = 1 + 2t + \frac{t^2}{4}$.

(a) Find the average velocity from $[1, 3]$.

$$\frac{s(3) - s(1)}{3 - 1} = 3$$

(b) Find the average velocity from $[1, 2]$.

$$\frac{s(2) - s(1)}{2 - 1} = 2.75$$

(c) Find the instantaneous velocity when $t = 1$.

$$\lim_{h\to 0} \frac{s(1+h) - s(1)}{h} \ \ldots \ = 2.5$$