MATH 150 Exam 7 Worksheets Fall

Worksheet for Section 1

1. Find the volume of the solid obtained by rotating the region bounded by $y = e^x,\ y = 0,\ x = 0,\ x = 1$ about the x-axis. Sketch the region and a typical disk or washer.

   the disks will have radius $r = e^x$ so $A = \pi(e^x)^2$

   $$V = \int_0^1 A(x)\,dx = \int_0^1 \pi(e^x)^2\,dx = \frac{\pi}{2}\left(e^2 - 1\right)$$

2. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x-1},\ x = 2,\ x = 5,\ y = 0$ about the x-axis. Sketch the region and a typical disk or washer.

   the disks will have radius $r = \sqrt{x-1}$ so $A = \pi(\sqrt{x-1})^2 = \pi(x-1)$

   $$V = \int_2^5 A(x)\,dx = \int_2^5 \pi(x-1)\,dx = \frac{15\pi}{2}$$

3. Find the volume of the solid obtained by rotating the region bounded by $x = y - y^2,\ x = 0$ about the y-axis. Sketch the region and a typical disk or washer.

   the disks will have radius $r = y - y^2$ so $A(y) = \pi(y - y^2)^2$

   $$V = \int_0^1 A(y)\,dy = \int_0^1 \pi(y - y^2)^2\,dy = \frac{\pi}{30}$$

Worksheet for Section 2

1. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x(x-1)^2$ and the x-axis about the y-axis. Sketch the region and a typical shell.

   the shells will have radius $r = x$ so the circumference is $2\pi x$ and the height is $x(x-1)^2$

   $$V = \int_0^1 2\pi x(x(x-1)^2)\,dx \;=\; \ldots \;=\; \frac{\pi}{15}$$

2. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $x = 4y^2 - y^3$ and the line $x = 0$ about the x-axis. Sketch the region and a typical shell.

   the shells will have radius $r = y$ so the circumference is $2\pi y$ and the height is $4y^2 - y^3$

   $$V = \int_0^4 2\pi y(4y^2 - y^3)\,dy \;=\; \ldots \;=\; \frac{512\pi}{5}$$

Worksheet for Section 3

1. Find the average value of $h(r) = \frac{3}{(1+r)^2}$ on the interval $[1, 6]$.

$$Avg \;=\; \frac{1}{6-1}\int_1^6 \frac{3}{(1+r)^2}\,dr \;=\; \frac{3}{5}\int_1^6 u^{-2}\,du \;=\; \dots\;\frac{3}{14}$$

Worksheet for Section 4

1. Find the length of $x = \frac{1}{3}\sqrt{y}(y-3), \qquad 1 \le y \le 9$

   HINT: Notice that the function is expressed as $x =$ stuff in y.

$$x = \frac{1}{3}y^{3/2} - y^{1/2} \;\text{ so }\; \frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2} \;=\; \frac{\sqrt{y}}{2} - \frac{1}{2\sqrt{y}} \;=\; \frac{y-1}{2\sqrt{y}}$$

$$\left(\frac{dx}{dy}\right)^2 = \left(\frac{y-1}{2\sqrt{y}}\right)^2 = \frac{y^2-2y+1}{4y}$$

$$\Longrightarrow \quad L = \int_1^9 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy \;=\; \int_1^9 \sqrt{1+\frac{y}{4}-\frac{1}{2}+\frac{1}{4y}}\,dy$$

$$= \int_1^9 \sqrt{\left(\frac{1}{2}y^{1/2}+\frac{1}{2}y^{-1/2}\right)^2}\,dy \;=\; \frac{1}{2}\left[\frac{2}{3}y^{3/2}+2y^{1/2}\right]\Bigg|_1^9 \;=\; \frac{32}{3}$$

Worksheet for Section 5

1. Find the area of the surface obtained by rotating the the curve $y = \sqrt[3]{x}$, $\quad 1 \leq y \leq 2$ about the y-axis.

$$\text{about the y-axis} \implies S = \int 2\pi x \; ds$$

$$y = \sqrt[3]{x} \implies y^3 = x \;\text{ so }\; 1 + \left(\frac{dx}{dy}\right)^2 = 1 + 9y^4$$

$$S = \int 2\pi x \; ds = \int_1^2 2\pi y^3 \sqrt{1+9y^4} \; dy = \frac{2\pi}{36} \int_1^2 \sqrt{1+9y^4} \cdot 36y^3 \; dy$$

$$\frac{\pi}{18} \left[\frac{2}{3} \left(1+9y^4\right)^{3/2} \Bigg|_1^2 \right] = \frac{\pi}{27} \left(145\sqrt{145} - 10\sqrt{10}\right)$$

2. Find the area of the surface obtained by rotating the the curve $x = 1 + 2y^2$, $\quad 1 \leq y \leq 2$ about the x-axis.

$$\text{about the x-axis} \implies S = \int 2\pi y \; ds$$

$$x = 1 + 2y^2 \implies y^3 = x \;\text{ so }\; 1 + \left(\frac{dx}{dy}\right)^2 = 1 + 16y^2$$

$$S = \int 2\pi y \; ds = \int_1^2 2\pi y \sqrt{1+16y^2} \; dy = \frac{\pi}{16} \int_1^2 \sqrt{1+16y^2} \cdot 32y \; dy$$

$$\frac{\pi}{16} \left[\frac{2}{3} \left(1+16y^2\right)^{3/2} \Bigg|_1^2 \right] = \frac{\pi}{24} \left(65\sqrt{65} - 17\sqrt{17}\right)$$