MATH 151 HW Problems Key Exam 4

Homework Key for Section 1

1. Which of the following functions are solutions of the differential equation $y'' + y = sin\ x$?

$$y = -\frac{1}{2}x\ cos\ x$$

2. Given the differential equation $y' = -y^2$:

   (a) What can you say about a solution by just looking at the differential equation?

   $$0\ or\ decreasing$$

   (b) Verify that all members of the family $y = 1/(x + C)$ are solutions.

   (c) Can you think of a solution that is not a member of the family in part (b)?

   $$y = 0$$

   (d) Find a solution to $y' = -y^2 \qquad y(0) = 0.5$

   $$y = \frac{1}{x+2}$$

3. A population is modeled by the differential equation

$$\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$$

   (a) For what values of P is the poulation increasing?

   $$0 < P < 4200$$

   (b) For what values of P is the poulation decreasing?

   $$P > 4200$$

   (c) What are the equilibrium solutions?

   $$P = 0, P = 4200$$

4. Explain why the functions with the given graphs can NOT be solutions to the following:

$$\frac{dy}{dt} = e^t(y-1)^2$$

Homework Key for Section 2

1. A direction field for the differential equation $y' = x\ sin\ y$ is shown.
   (a) Sketch the graphs of the solutions that satisfy the given initial conditions.
      i. $y(0) = 1$
      ii. $y(0) = 2$
      iii. $y(0) = \pi$
      iv. $y(0) = 4$
      v. $y(0) = 5$
   (b) Find all of the equilibrium solutions.

$$y = 0, y = \pi$$

2. Given the following direction fields, match them with their differential equations.
   (a) $y' = 2 - y$ top left
   (b) $y' = x(2 - y)$ top right
   (c) $y' = x + y - 1$ bottom left
   (d) $y' = sin\ x\ sin\ y$ bottom right
3. Use Euler's method with step size 0.1 to estimate $y(0.5)$ where y is the solution to $y' = y + xy \quad , \quad y(0) = 1$

$$y(0.5) \approx 1.7616$$

Homework Key for Section 3

1. Solve the following differential equations.

   (a)
   $$\frac{dy}{dx} = \frac{y}{x}$$
   $$y = Kx$$

   (b)
   $$(x^2+1)y' = xy$$
   $$y = K\sqrt{x^2+1}$$

   (c)
   $$(1 + tan\ y)y' = x^2 + 1$$
   $$y + ln \mid sec\ y \mid = \frac{x^3}{3} + x + C$$

   (d)
   $$\frac{dy}{dt} = \frac{te^t}{y\sqrt{1+y^2}}$$
   $$y = \pm\sqrt{[3(te^t - e^t + C)]^{2/3} - 1}$$

   (e)
   $$\frac{dy}{dx} = \frac{x}{y} \quad , \quad y(0) = -3$$
   $$y = -\sqrt{x^2+9}$$

   (f)
   $$x\ cos\ x = (2y + e^{3y})y' \quad , \quad y(0) = 0$$
   $$cos\ x + x\ sin\ x = y^2 + \frac{e^{3y}}{3} + \frac{2}{3}$$

2. Given the following, $\frac{dP}{dt} = k(M - P)$, where $P(t)$ is the performance after training time t, M is the maximum level of performance and k is a positive constant. Solve this differential equation for P and find the limit of that expression.

$$P(t) = M - Me^{-kt} \quad , \quad M$$

Homework Key for Section 4

1. Suppose a population develops according to the logistic equation

$$\frac{dP}{dt} = 0.05P - 0.0005P^2$$

where t is measured in weeks.

(a) What is the carrying capacity?

100

(b) Given the direction field, sketch solutions for initial populations of 20,40,60,80,120 and 140.

(c) What are the equilibrium solutions?

$$P = 0\ ,\ P = 100$$

Homework Key for Section 5

1. Determine whether the following differential equations are linear.

   (a) $y' + cos\ x = y$

   $$yes$$

   (b) $yy' + xy = x^2$

   $$no$$

2. Solve the following:

   (a) $y' + 2y = 2e^x$

   $$y = \frac{2}{3}e^x + Ce^{-2x}$$

   (b) $xy' - 2y = x^2$

   $$y = x^2\ ln \mid x \mid + Cx^2$$

   (c) $xy' + y = \sqrt{x}$

   $$y = \frac{2}{3}\sqrt{x} + \frac{C}{x}$$

   (d) $y' = x + y \quad , \quad y(0) = 2$

   $$y = -x - 1 + 3e^x$$

   (e) $\frac{dv}{dt} - 2tv = 3t^2e^{t^2} \quad , \quad v(0) = 5$

   $$v = t^3e^{t^2} + 5e^{t^2}$$