MATH 150 Review Key Exam 7

1. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

   (a) $y = 2x, \quad y = x^2, \quad$ about the x-axis

   $A(x) = \pi((2x)^2) - \pi(x^2)^2$, so

   $$\mathbf{V} = \pi \int_0^2 (4x^2 - x^4)\, dx = \frac{64\pi}{15}$$

   (b) $x = 0, \quad x = 9 - y^2, \quad$ about $x = -1$

   $A(y) = \pi\{[(9 - y^2) - (-1)]^2 - [0 - (-1)]^2\}$, so

   $$\mathbf{V} = \pi \int_{-3}^3 [(10 - y^2)^2 - 1]\, dy = \frac{1656\pi}{5}$$

2. Use shells to find the volume of the solid obtained by rotating the region bounded by $x = 1 + y^2, x = 0, y = 1$ and $y = 2$ about the x axis.

   $$\int_1^2 2\pi y(1 + y^2)\, dy \ \ldots = \frac{21\pi}{2}$$

3. The height of a monument is 20 meters. A horizontal cross section at a distance x meters from the top is an equilateral triangle with side $\frac{x}{4}$ meters. Find the volume of the monument.

   Recall that volume is the integral of cross-sectional area, so..

   $A_{cross\ section} = \frac{1}{2}bh = \frac{1}{2}(\frac{x}{4})(\frac{\sqrt{3}x}{8}) \ \ldots$

   $$\int_0^{20} \frac{\sqrt{3}x^2}{64}\, dx \ \ldots = \frac{125\sqrt{3}}{3}$$

4. Find the average value of $f(x) = (x - 3)^2$ on $[2, 5]$.

   $$\tfrac{1}{3}\int_2^5 (x - 3)^2\, dx = 1$$

5. Find c such that $f_{ave} = f(c)$ for the previous problem.

   When is f(x) = 1? At $x = 2$ and $x = 4$

6. Find the length of the following curve. $y = ln(sec\ x),\ 0 \le x \le \pi/4$

   $$\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx \ \ldots = \ln(\sqrt{2} + 1)$$

7. Find the area of the surface obtained by rotating $y = cosh\ x,\ 0 \le x \le 1$ about the x axis.

   $$\int_0^1 2\pi y \sqrt{1 + \left[\frac{d}{dx}(cosh\ x)\right]^2}\, dx = \int_0^1 2\pi(cosh\ x)\sqrt{[cosh\ x)]^2}\, dx = \pi[1 + \frac{1}{4}(e^2 - e^{-2})]$$

8. Find the area of the surface obtained by rotating $x = \frac{1}{3}(y^2+2)^{3/2},\ 1 \le y \le 2$ about the x axis.

$$\int_1^2 2\pi y\sqrt{y^4+2y^2+1}\,dy = \ \ldots\ = \frac{21\pi}{2}$$