MATH 151 Exam 5 Review Key Spring

1. Given $x = t^2 - 2$, $y = 5 - 2t$ eliminate the parameter to find a Cartesian equation.

$$x = \left[\frac{1}{2}(5-y)\right]^2 - 2 \quad \text{or} \quad x = \frac{1}{4}(5-y)^2 - 2$$

2. Given $x = 1 + 3t^2$, $y = 4 + 2t^3$ find the length of the curve.

$$\frac{dx}{dt} = 6t \quad \text{and} \quad \frac{dy}{dt} = 6t^2 \quad \Longrightarrow$$

$$L = \int_a^b \sqrt{36t^2 + 36t^4}\; dt \;=\; \int_a^b 6t\sqrt{1+t^2}\; dt \;=\; 3\left[\frac{2}{3}u^{3/2}\Big|_a^b\right]$$

3. Find the slope of the tangent line to the curve $r = 1 + cos\ \theta$

$$x = cos\ \theta + cos^2\ \theta \quad \text{and} \quad y = sin\ \theta + sin\ \theta\ cos\ \theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{cos\ \theta + cos\ 2\theta}{-sin\ \theta - sin\ 2\theta}$$

4. Find the area enclosed by one loop of $r = 2\ cos\ 4\theta$

$$A = \int_0^{\pi/8} \frac{1}{2}(2cos\ 4\theta)^2\ d\theta = 2\int_0^{\pi/8} (1 + cos\ 8\theta)\ d\theta$$

$$= 2\left[\theta + \frac{1}{8}sin\ 8\theta\right]\Big|_0^{\pi/8} = \frac{\pi}{4}$$

5. Find the surface area generated by rotating $x = e^t - t$, $y = 4e^{t/2}$, $0 \le t \le 1$ about the y-axis.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{2t} - 2e^t + 1 + 4e^t = (e^t + 1)^2$$

$$SA = \int_0^1 2\pi(e^t - 1)(e^t + 1)\ dt = \ldots = \pi e^2 + 2\pi e - 6\pi \;\approx\; 21.44$$

6. For which values of t is the curve concave upward? $x = 4 + t^2$, $y = t^2 + t^3$

$$\frac{dy}{dx} = 1 + \frac{3}{2}t \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{3}{4t} \quad \Longrightarrow \quad CU \quad \text{when} \quad t > 0$$

7. Find the length of the curve. $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 3$

$$L = \int_0^3 \sqrt{e^{2t} + 2 + e^{-2t}}\ dt = \int_0^3 \sqrt{(e^t + e^{-t})^2}\ dt = e^3 - e^{-3}$$

8. Find the area of the region enclosed by one loop of the curve. $r = 4\ sin\ 3\theta$

$$A = \int_0^{\pi/3} \frac{1}{2}(4sin\ 3\theta)^2\ d\theta \ldots = \frac{4\pi}{3}$$

9. Find the exact length of the polar curve. $r = 3\ sin\ \theta$, $0 \le \theta \le \pi/3$

$$L = \int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta \ldots = \pi$$