MATH 150 Exam 5 Review Fall

1. Let $f(x) = x^4 - 4x^3$.

   (a) Find the intervals where $f(x)$ is increasing AND decreasing.

   increasing on $(3, \infty)$ decreasing on $(-\infty, 3)$

   (b) Find the local maximum and local minimum values.

   $x = 3$ is a local min

   (c) Find the intervals of concavity.

   CU on $(-\infty, 0)$ and $(2, \infty)$

   CD on $(0, 2)$

   (d) Find the inflection points.

   IP at $x = 0$, and $x = 2$

2. A number a is called a fixed point of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x, then f has at most one fixed point.

   Suppose you have two fixed points $f(a) = a$ and $f(b) = b$ apply the MVT to (a, b)

   then $f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{b - a}{b - a} = 1$ which gives us a contradiction

3. If $f(x) = x^4 - 2x^2 + 3$

   (a) Find the intervals where f is increasing and decreasing

   increasing on $(-1, 0)$ and $(1, \infty)$ decreasing on $(-\infty, -1)$ and $(0, 1)$

   (b) Find any local max and min

   $x = -1, 1$ is a local min $x = 0$ is a local max

(c) Find the intervals of concavity and inflection points

$$\text{CU on } \left(-\infty, -\sqrt{\frac{1}{3}}\right) \text{ and } \left(\sqrt{\frac{1}{3}}, \infty\right)$$

$$\text{CD on } \left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$$

$$\text{IP at } x = -\sqrt{\frac{1}{3}} \text{ and } x = \sqrt{\frac{1}{3}}$$

4. Use the Mean Value Theorem to establish the inequality $e^x > 1 + x$ for $x > 0$.

apply the MVT to $(0, x)$

$$\text{then } f'(c) = e^c = \frac{f(x) - f(0)}{x} = \frac{e^x - 1}{x} \implies$$
$$xe^c = e^x - 1 \implies xe^c + 1 = e^x \implies e^x - 1 > x$$

5. If $f(x) = 2x^3 - 3x^2 - 12x$

(a) Find the intervals where f is increasing and decreasing

increasing on $(-\infty, -1)$ and $(2, \infty)$ decreasing on $(-1, 2)$

(b) Find any local max and min

$x = 2$ is a local min $x = -1$ is a local max

(c) Find the intervals of concavity and inflection points.

$$\text{CU on } \left(\frac{1}{2}, \infty\right) \text{ CD on } \left(-\infty, \frac{1}{2}\right)$$

$$\text{IP at } x = \frac{1}{2}$$

6. A poster is to have an area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?

$$\text{max } = xy \implies 180 = (y+3)(x+2)\ldots x = \frac{180}{y+3} - 2$$

$$\ldots x = \sqrt{270} - 3 \approx 13.4 \quad y = \frac{180}{\sqrt{270}} - 2 \approx 8.95$$

7. Use Newton's Method to find the root of the equation $x^5 - x^4 + 3x^2 - 3x - 2 = 0$ in the interval $[1, 2]$ correct to six decimal places.

$$x_5 \approx x_6 \approx 1.297383$$

8. A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal? (**hint:** surface area of hemisphere is $(1/2)4\pi r^2$)

$$r = h = \sqrt[3]{\frac{3V}{5\pi}}$$

9. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

$$x = 500\ ,\ \ y = 125$$

10. Find f for the following:

(a) $f'(x) = e^x - \frac{2}{\sqrt{x}},\ \ f(0) = -1$

$$f = e^x - 4\sqrt{x} - 2$$

(b) $f'(t) = 2t - 3sin\ t,\ \ f(0) = 5$

$$f = t^2 + 3cos\ t + 2$$

(c) $f'(u) = \frac{u^2+\sqrt{u}}{u},\ \ f(1) = 3$

$$f = \frac{1}{2}u^2 + 2\sqrt{u} + \frac{1}{2}$$

(d) $f''(x) = 1 - 6x + 48x^2,\ \ f(0) = 1,\ \ f'(0) = 2$

$$f = \frac{1}{2}x^2 - x^3 + 4x^4 + 2x + 1$$

(e) $f'(x) = 2/(1 + x^2),\ \ f(0) = -1$

$$f = 2arctan\ x - 1$$

(f) $f''(x) = 2x^3 + 3x^2 - 4x + 5,\ \ f(0) = 2,\ \ f(1) = 0$

$$f = \frac{x^5}{10} + \frac{x^4}{4} - \frac{2}{3}x^3 + \frac{5}{2}x^2 - \frac{250}{61}x + 2$$