MATH 151 HW Problems Key Exam 3

Homework Key for Section 1

1. Determine whether the geometric series is C or D. If C, find the sum.

   (a) $3 + 2 + \frac{4}{3} + \frac{8}{9} + \ldots$

   $$9$$

   (b) $3 - 4 + \frac{16}{3} - \frac{64}{9} + \ldots$

   $$D$$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

   $$\frac{1}{7}$$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

   $$D$$

2. Determine whether the series is C or D. If C, find the sum.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{n^2 - 1}$

   $$D$$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n}$

   $$\frac{5}{2}$$

   (c) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{2}$

   $$D$$

   (d) $\displaystyle\sum_{n=1}^{\infty} ln\left(\frac{n^2 + 1}{2n^2 + 1}\right)$

   $$D$$

   (e) $\displaystyle\sum_{n=1}^{\infty} arctan\ n$

   $$D$$

   (f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right)$

   $$\frac{e}{e-1}$$

3. Determine whether the telescoping series is C or D. If C, find the sum.

(a) $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

$$\frac{11}{6}$$

4. Find the values of x for which the series converges. Then find the sum for those values.

(a) $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

$$-3 < x < 3 \quad \frac{x}{3-x}$$

(b) $\sum_{n=0}^{\infty} 4^n\ x^n$

$$-\frac{1}{4} < x < \frac{1}{4} \quad \frac{1}{1-4x}$$

5. The **Cantor Set** is constructed as follows. Start with the closed interval $[0,1]$ and remove the open interval $(\frac{1}{3}, \frac{2}{3})$. That leaves the two intervals $[0, \frac{1}{3}]$ and $[\frac{2}{3}, 1]$. Remove the open middle third of each of those. Continue with this process indefinitely. The Cantor Set consists of those numbers that remain after this process, $0, \frac{1}{3}, \frac{2}{3}, 1, \ldots$.

(a) How many numbers are in the Cantor Set? (An infinite number)

(b) Show that the total length of all of those numbers removed is 1. (Use a geometric series)

Homework Key for Section 2

1. Determine whether the series is C or D.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$

   C

   (b) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

   C

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$

   D

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3}$

   C

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

   D

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

   C

   (g) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ ln\ n}$

   D

Homework Key for Section 3

1. Determine whether the series converges or diverges.

   (a) $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

   C

   (b) $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$

   D

   (c) $\sum_{n=1}^{\infty} \frac{cos^2\ n}{n^2+1}$

   C

   (d) $\sum_{n=1}^{\infty} \frac{n-1}{n\ 4^n}$

   C

   (e) $\sum_{n=1}^{\infty} \frac{arctan\ n}{n^{1.2}}$

   C

   (f) $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$

   C

   (g) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

   D

   (h) $\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$

   D

2. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is convergent. Show that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$

then $\sum a_n$ is also convergent. Use this fact to show the following converges:

$$\sum_{n=1}^{\infty} \frac{ln\ n}{n^3}$$

3. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is divergent. Show that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$$

then $\sum a_n$ is also divergent. Use this fact to show the following diverges:

$$\sum_{n=2}^{\infty} \frac{1}{ln\ n}$$

Homework Key for Section 4

1. Determine whether the series is C or D.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

   C

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

   D

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$

   C

   (d) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{ln\ n}$

   D

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{cos(n\pi)}{n^{3/4}}$

   C

2. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$ is convergent. How many terms do you need to add so that $|\text{ error }| < 0.00005$?

   $5\ terms$

Homework Key for Section 5

1. Determine whether the series is AC, CC or D.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$

   AC

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$

   CC

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3}$

   AC

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$

   AC

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ arctan\ n}{n^2}$

   AC

   (f) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{ln\ n}$

   CC

   (g) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

   AC

Homework Key for Section 6

1. Determine whether the series is C or D.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+3^n}$

   C

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$

   D

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2\ 2^{n-1}}{(-5)^n}$

   C

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{ln\ n}}$

   D

   (e) $\displaystyle\sum_{n=1}^{\infty} n^2\ e^{-n}$

   C

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n\ n^2}{n!}$

   C

   (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$

   D

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

   C

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

   C