MATH 150 HW Problems Key Exam 2

Homework for Section 1

1. Sketch a graph of an example of a function f that satisfies all of the given conditions

   (a) $\lim_{x\to 3^+} f(x) = 4, \quad \lim_{x\to 3^-} f(x) = 2, \quad \lim_{x\to -2} f(x) = 2, \quad f(3) = 3, \quad f(-2) = 1$

   (b) $\lim_{x\to 0^-} f(x) = 1, \quad \lim_{x\to 0^+} f(x) = -1, \quad \lim_{x\to 2^-} f(x) = 0, \quad \lim_{x\to 2^+} f(x) = 1, \quad f(2) = 1, \quad f(0)\ is\ undefined$

2. Use a table to estimate the value of

$$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$$

3. Find the following limits if they exist:

   (a)

$$\lim_{x\to 5^+} \frac{6}{x-5}$$

$$\infty$$

   (b)

$$\lim_{x\to 1} \frac{2-x}{(x-1)^2}$$

$$\infty$$

4. In the theory of relativity the mass of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

   where m_0 is the rest mass and c is the speed of light. What happens as $v \longrightarrow c^-$?

$$\infty$$

Homework for Section 2

1. Evaluate the following limit by JUSTIFYING EVERY STEP $\lim_{x\to 2} \frac{2x^2+3}{x^2+2x-1}$

2. Is the following a true statement?

$$\frac{x^2+x-6}{x-2} = x+3$$

3. Why is this statement true then?

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} x+3$$

4. Find the following limits if they exist:

   (a) $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$...5

   (b) $\lim_{x\to 2} \frac{x^2+x+6}{x-2}... \pm\infty$

   (c) $\lim_{x\to -3} \frac{x^2-9}{2x^2+7x+3}$...6/5

   (d) $\lim_{h\to 0} \frac{(4+h)^2-16}{h}$...8

   (e) $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$...3/2

   (f) $\lim_{t\to 9} \frac{9-t}{3-\sqrt{t}}$...6

   (g) $\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7}$...1/6

   (h) $\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}... -1/16$

   (i) $\lim_{x\to -4} \mid x+4 \mid$...0

   (j) $\lim_{x\to 4} \frac{|x-4|}{x-4}...DNE$

5. Let $f(x) = \begin{cases} 4-x^2 & if \quad x \le 2 \\ x-1 & if \quad x > 2 \end{cases}$

   (a) Find $\lim_{x\to 2^-} f(x)$...0

   (b) Find $\lim_{x\to 2^+} f(x)$...1

   (c) Find $\lim_{x\to 2} f(x)...DNE$

6. If $f(x) = \begin{cases} x^2 & if\ x\ is\ rational \\ 0 & if\ x\ is\ irrational \end{cases}$ prove $\lim_{x\to 0} f(x) = 0$

   use the squeeze theorem

7. Is there a number a such that

$$\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2}$$

   exists? If so, find a and the limit.

   If the bottom goes to 0 then the top has to as well

Homework for Section 3

1. Graph $f(x) = \sqrt{x}$ and use it to find a number δ such that $| \sqrt{x} - 2 | < 0.4$ whenever $| x - 4 | < \delta$

2. Prove the following statements using the ϵ, δ definition.

   (a) $\lim_{x \to -3} (1 - 4x) = 13$

   (b) $\lim_{x \to 4} (7 - 3x) = -5$

   (c) $\lim_{x \to 3} \frac{x}{5} = \frac{3}{5}$

   (d) $\lim_{x \to -2} (x^2 - 1) = 3$

3. If $f(x) = \begin{cases} 0 & if\ x\ is\ rational \\ 1 & if\ x\ is\ irrational \end{cases}$ prove $\lim_{x \to 0} f(x)$ does not exist.

Homework for Section 4

1. Sketch the graph of a function that is continuous everywhere except at $x = 2$

2. Use the definition to show that $f(x) = x^2 + \sqrt{7 - x}$ is continuous at 4.

3. Explain why the following functions are discontinuous at the given number a

   (a) $f(x) = ln \ \mid x - 2 \mid$ $\quad a = 2$*...not in domain*

   (b) $f(x) = \begin{cases} \frac{1}{x-1} & if \ \ x \neq 1 \\ 2 & if \ \ x = 1 \end{cases}$ $\quad a = 1$*...limit does not equal* $f(1)$

   (c) $f(x) = \begin{cases} e^x & if \ \ x < 0 \\ x^2 & if \ \ x \geq 0 \end{cases}$ $\quad a = 0$*...limit DNE*

4. Find the numbers where the following functions are discontinuous

   (a) $f(x) = \begin{cases} 1 + x^2 & if \ \ x \leq 0 \\ 2 - x & if \ \ 0 < x \leq 2 \\ (x-2)^2 & if \ \ x > 2 \end{cases}$ *...at* 0

   (b) $f(x) = \begin{cases} x + 2 & if \ \ x < 0 \\ e^x & if \ \ 0 \leq x \leq 1 \\ 2 - x & if \ \ x > 1 \end{cases}$ *...at* 0 *and* 1

5. For what constant c is the following function continuous on $(-\infty, \infty)$. $f(x) = \begin{cases} cx + 1 & if \ \ x \leq 3 \\ cx^2 - 1 & if \ \ x > 3 \end{cases}$

   $$c = 1/3$$

6. If $f(x) = x^3 - x^2 + x$ show there is a number c such that $f(c) = 10$.

   Use IVT

7. Use the IVT to show that $f(x) = x^4 + x - 3 = 0$ must have a root in $(1, 2)$

8. For what values is the following function continuous? $f(x) = \begin{cases} 0 & if \ x \ is \ rational \\ 1 & if \ x \ is \ irrational \end{cases}$

   none

Homework for Section 5

1. Sketch the graph of an example of a function that satisfies the given conditions.

   (a) $\lim_{x\to 0^+} f(x) = \infty, \quad \lim_{x\to 0^-} f(x) = -\infty, \quad \lim_{x\to\infty} f(x) = 1, \quad \lim_{x\to -\infty} f(x) = 1$

   (b) $\lim_{x\to 0^-} f(x) = -\infty, \quad \lim_{x\to 0^+} f(x) = \infty, \quad \lim_{x\to 2} f(x) = -\infty, \quad \lim_{x\to\infty} f(x) = \infty, \quad \lim_{x\to -\infty} f(x) = 0$

2. Find the following limits if they exist:

   (a)
$$\lim_{x\to\infty} \frac{3x+5}{x-4}$$
$$3$$

   (b)
$$\lim_{x\to\infty} \frac{2-3x^2}{5x^2+4x}$$
$$-3/5$$

   (c)
$$\lim_{x\to -\infty} \frac{x^2+2}{x^3+x^2-1}$$
$$0$$

   (d)
$$\lim_{u\to\infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)}$$
$$2$$

   (e)
$$\lim_{x\to\infty} \frac{\sqrt{9x^6-x}}{x^3+1}$$
$$3$$

   (f)
$$\lim_{x\to\infty} \; cos\ x$$
$$DNE$$

   (g)
$$\lim_{x\to\infty} \sqrt{x}$$
$$\infty$$

3. Find the horizontal and vertical asymptotes of $y = \frac{x^2+4}{x^2-1}$...*horiz at* 1 *vert at* $1, -1$

Homework for Section 6

1. Find the slope of the tangent line at the given point for the following functions:
   (a) $f(x) = x^2 + 2x$ at $(-3, 3) ... -4$
   (b) $f(x) = \sqrt{2x+1}$ at $(4, 3) ... 1/3$
   (c) $f(x) = \frac{1}{x+1}$ at $(1, 1/2) ... -1/4$
2. A ball is thrown into the air so that its height in feet after t seconds is $s(t) = t^2 - 8t + 18$
   (a) What is the average velocity over the following intervals?
      i. $[3, 4]$
      ii. $[3.5, 4]$
      iii. $[4, 5]$
      iv. $[4, 4.5]$
   (b) What is the instantaneous velocity at $t = 4$?
3. A different ball is thrown into the air so that its height in feet after t seconds is $h(t) = 58t - 0.83t^2$
   (a) What is the velocity after one second?
   (b) When will the ball strike the ground?
   (c) What is the velocity at this time?
4. Sketch a graph of a function f that satisfies all of the given conditions:
   (a) $f(0) = 0,\ f'(0) = 3,\ \ f'(1) = 0,\ \ and\ \ f'(2) = -1$
   (b) $f(0) = 0,\ f'(0) = 3,\ \ f'(1) = 0,\ \ and\ \ f'(2) = 1$
5. Find $f'(a)$ for the following functions:
   (a) $f(x) = x^2 - 2x + 2 ... 2a - 2$
   (b) $f(x) = \frac{2x+1}{x+3}$

   $$\frac{5}{(a+3)^2}$$

   (c) $f(x) = \sqrt{3x+1}$

   $$\frac{3}{2\sqrt{3a+1}}$$

6. Each limit represents the derivative for some f at some number a. Find f and a for each.
   (a) $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h} ... x^{10}\ \ a = 1$
   (b) $\lim_{x\to 5} \frac{2^x-32}{x-5} ... 2^x\ \ a = 5$
   (c) $\lim_{x\to \pi/4} \frac{tan\ x-1}{x-\pi/4} ... tan\ x\ \ a = \pi/4$
7. Find the velocity when $t = 2$ if the displacement is given by $s(t) = t^2 - 6t - 5 .... -2$

8. The number of bacteria after t hours is $n = f(t)$.

   (a) What is meant by $f'(5)$?

   (b) Given unlimited space and nutrients, which is larger, $f'(5)$ or $f'(10)$?

   (c) Given limited space and nutrients, would you change your answer? Why?

Homework for Section 7

1. Make a careful sketch of the following functions and on the same set of axes sketch f'. Can you guess a formula for $f'(x)$ from the graphs?

   (a) $f(x) = sin\ x$

   (b) $f(x) = ln\ x$

   (c) $f(x) = e^x$

2. The following table gives displacements at various times t.

t	$s(t)$	t	$s(t)$
1	68	6	54
2	75	7	49
3	69	8	45
4	61	9	42
5	56	10	40

   (a) What is the meaning of $s'(t)$?

   (b) What are its units?

   (c) Construct a table of values for $s'(t)$.

3. SHOW where $f(x) = \mid x - 3 \mid$ is not differentiable.