MATH 245 Logic Worksheet Spring

1. Evaluate each proposition if $p = F, \quad q = T, \quad r = F$:
   (a) $p \vee q$
   (b) $\overline{p} \vee \overline{(q \wedge r)}$

2. Write the truth table for each:
   (a) $p \wedge \overline{q}$
   (b) $(p \wedge q) \wedge \overline{p}$
   (c) $(p \vee q) \wedge (\overline{p} \vee q) \wedge (p \vee \overline{q}) \wedge (\overline{p} \vee \overline{q})$

3. * Write truth tables to verify BOTH of De Morgan's laws.

4. * Write a truth table to verify the contrapositive.

MATH 245 Induction Worksheet Spring

1. Use Induction to show

$$\frac{1}{1(3)} + \frac{1}{3(5)} + \frac{1}{5(7)} + \ \dots \ + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

2. Observe that:

$$1 = 1^2$$
$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$
$$1 + 3 + 5 + 7 = 4^2$$

Figure out a general formula and use Induction to show it is correct.

3. * Prove by Induction that $7^n - 4^n$ is a multiple of $3 \ \forall \ n \in \mathbb{N}$

MATH 245 Set Theory Worksheet Spring

1. Let $U = \{1, 2, 3, \ldots, 10\}$, $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{2, 4, 6, 8\}$. Find:

   (a) $A \cup B$

   (b) $B \cap C$

   (c) $A - B$

   (d) $B \cap (C - A)$

2. For each condition, what relation must hold between A and B.

   (a) $A \cap B = A$

   (b) $A \cap B = B$

3. Among a group of 165 students, 8 are taking calculus, psychology and French; 33 are taking calculus and French; 20 are taking calculus and psychology; 24 are taking psychology and French; 79 are taking calculus; 83 are taking psychology; and 63 are taking French. Use a Venn diagram to find out how many are taking none of those subjects.

4. * Show that $\mid A \cup B \mid = \mid A \mid + \mid B \mid - \mid A \cap B \mid$

5. * Prove DeMorgan's Laws for sets using Venn Diagrams.

6. * Prove by Induction that if $\mid X \mid = n$, then $\mid \mathcal{P}(X) \mid = 2^n$

MATH 245 Relations Worksheet Spring

1. Let $X = \{1, 2, 3, 4, 5\}$. Define the relation R by the rule $(x, y) \epsilon \ R \ if \ 3 \ divides \ x - y$. List the elements of R.

2. Repeat the previous exercise if the rule is $(x, y) \epsilon \ R \ if \ x + y \leq 6$.

3. * Determine if the following relations are reflexive, transitive and symmetric.

   (a) $(x, y) \epsilon \ R \ if \ x = y$.

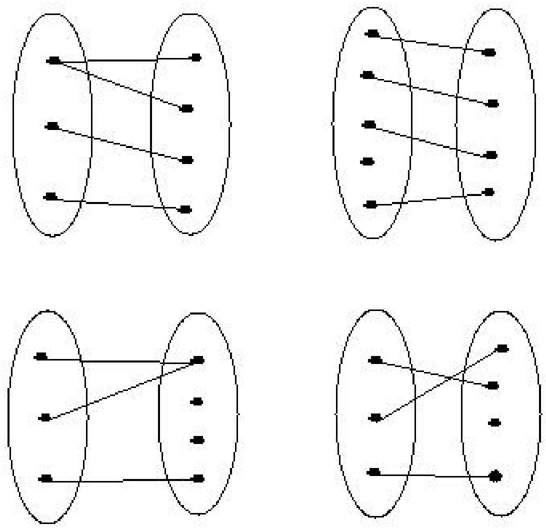
   (b) $(x, y) \epsilon \ R \ if \ x < y$.

4. * Let $X = \{1, 2, 3, 4, 5\}$. Determine if the following relation is an equivalence relation. If it is, list the equivalence classes.

$$\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (3, 1)\}$$

MATH 245 Functions Worksheet Spring

1. Consider the pictures below. Classify each as being a function, and if so, as surjective, injective or bijective.



2. * Given $f : A \longmapsto B$ and $y \in B$, under what conditions on f can we assert that there exists an x in A such that $f(x) = y$?

3. * Given $f : A \longmapsto B$ and $y \in B$, under what conditions on f can we assert that there exists a unique x in A such that $f(x) = y$?

4. * For the following function, $f(x) = x^2 + 2$, $f : \mathbb{R} \longmapsto \mathbb{R}$, find:

   (a) the codomain

   (b) the range

   (c) the domain

   (d) Is f injective?

   (e) Is f surjective?

MATH 245 Cardinality Worksheet Spring

1. Find a bijection, $f : \mathbb{N} \longmapsto \mathbb{Z}$, thereby showing that the set $\mathbb{Z}$ of all integers is also denumerable.

2. * Show that the set of rational numbers, $\mathbb{Q}$, is countable.

3. * Show that the set of irrational numbers is uncountable.

MATH 245 Number Theory Worksheet 1 Spring

1. Find $(51, 34)$ and $[51, 34]$.

2. Find all $d > 0$ such that $18 \mid d$ and $d \mid 216$.

3. For what integers a is $1 \mid a$ true?

4. For what integers a is $a \mid 0$ true?

5. Find q and r by the Division Algorithm if $a = 13$ and $b = 380$

6. * Prove that if $d \mid a$, $d \mid b$ and $d \mid c$, and if x, y and z are any integers, then d divides $ax + by + cz$.

7. * Show why $ab/(a, b)$ must be an integer whenever (a, b) is defined.

8. * Show that $[a, b]$ is defined if and only if neither a nor b is 0.

MATH 245 Number Theory Worksheet 2 Spring

1. Find (a, b) using the Euclidean Algorithm and solve backwards to get an x and y such that $ax + by = (a, b)$ for $a = 217$, $b = 341$.

2. Use the information from 1 to solve $217x + 341y = 62$.

3. Why is $4x + 6y = 25$ unsolvable?

4. Does $21x - 14y = 10000$ have a solution? Why or why not?

5. Find all solutions to $5x + 6y = 100$.

6. * Prove that if v is a linear combination of w and x and if w and x are each linear combinations of y and z then v is a linear combination of y and z.

7. * Farmer A owes Farmer B \$10. Neither has any cash, but Farmer A has 14 cows that are valued at \$185 each. Farmer A suggests paying his debt in cows with Farmer B making change by giving A some pigs valued at \$110 each. Is this possible, and how?

MATH 245 Number Theory Worksheet 3 Spring

1. Use the Sieve of Eratosthenes to find all the primes between 1000 and 1025.

2. Find $d(900)$.

3. * Show that if $d \mid a$ and $e \mid b$, then $de \mid ab$.

4. * Prove that if $(a, b) > 1$ then $d(ab) < d(a)d(b)$.

5. * Show that if $d(n)$ is prime, then n is a power of a prime.