MATH 151 Exam 1 Review KEY Spring

Evaluate the following integrals:

1. $\int ln(2x+1)\ dx$

   if we let $u = ln(2x+1)$ *and* $dv = dx$ *then parts gives us* $x\ ln(2x+1) - \int \frac{2x}{2x+1}\ dx$
   $= x\ ln(2x+1) - \int \left(1 - \frac{1}{2x+1}\right)\ dx\ =\ x\ ln(2x+1) - x + \frac{1}{2}ln(2x+1) + C$

2. $\int t\ sin\ 2t\ \ dt$

   again parts works with $u = t$ *and* $dv = sin\ 2t\ dt$ $\ldots\ =\ -\frac{t}{2}\ cos\ 2t + \frac{1}{2}\int cos\ 2t\ \ dt$
   $=\ -\frac{t}{2}\ cos\ 2t + \frac{1}{4}\ sin\ 2t + C$

3. $\int_0^{\pi/2} cos^5 x\ \ dx$

   $= \int_0^{\pi/2} (cos^2\ x)^2\ cos\ x\ \ dx\ =\ \int_0^1 (1-u^2)^2\ \ du\ =\ \ldots\ =\ \frac{8}{15}$

4. $\int tan^3(2x)\ sec^5(2x)\ \ dx$

   $\int tan^2(2x)\ \ sec^4(2x)\ \ sec(2x)\ \ tan(2x)\ \ dx\ =\ \frac{1}{2}\int (u^2-1)u^4\ \ du\ =\ \frac{1}{14}\ sec^7(2x) - \frac{1}{10}\ sec^5\ (2x) + C$

5. $\int x^3\sqrt{9-x^2}\ \ dx$

   if we let $x = 3sin\ \theta$ *and* $dx = 3cos\ \theta\ d\theta\ \Longrightarrow\ \int 3^3\ sin^3\ \theta\ 3\ cos\ \theta\ 3\ cos\ \theta\ \ d\theta$
   $= 3^5 \int sin^3\ \theta\ cos^2\ \theta\ \ d\theta\ =\ -3^5 \int (1-u^2)u^2\ \ du = \ \ldots\ =\ 3^5\left[\frac{1}{5}\ \frac{(9-x^2)^{5/2}}{3^5} - \frac{1}{3}\ \frac{(9-x^2)^{3/2}}{3^3}\right] + C$

6. $\int \frac{t^5}{\sqrt{t^2+2}}\ \ dt$

   if we let $t = \sqrt{2}\ tan\ \theta$ *and* $dt = \sqrt{2}\ sec^2\ \theta\ d\theta\ =\ \ldots\ =\ \frac{1}{15}\ \sqrt{t^2+2}\ (3t^4 - 8t^2 + 32) + C$

7. $\int_0^1 \frac{2x+3}{(x+1)^2}\ \ dx$

   $= \int_0^1 \left[\frac{2}{x+1} + \frac{1}{(x+1)^2}\right]\ \ dx =\ 2\ ln\ 2 + \frac{1}{2}$

8. $\int \frac{dx}{x^4 - x^2}$

$$= \int \left[\frac{-1}{x^2} + \frac{1/2}{x-1} - \frac{1/2}{x+1} \right] dx = \frac{1}{x} + \frac{1}{2}\, ln \left| \frac{x-1}{x+1} \right| + C$$