MATH 151 Worksheet Keys Exam 2

Worksheet for Section 1

1. Evaluate the following improper integrals:

(a) $\displaystyle\int_1^\infty \frac{1}{(3x+1)^2}\ dx$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{(3x+1)^2}\ dx$$

$$\textit{so} \quad \int \frac{1}{(3x+1)^2}\ dx = \frac{1}{3}\int \frac{1}{u^2}\ du = -\frac{1}{3(3x+1)} + C$$

$$\textit{so} \quad \lim_{t\to\infty}\left(-\frac{1}{3(3x+1)}\Bigg|_1^t\right) = \lim_{t\to\infty}\left[-\frac{1}{3(3t+1)} + \frac{1}{12}\right] = \frac{1}{12} \quad \textit{so convergent}$$

(b) $\displaystyle\int_{-2}^3 \frac{1}{x^4}\ dx$

$$= \int_{-2}^0 \frac{1}{x^4}\ dx + \int_0^3 \frac{1}{x^4}\ dx$$

$$\textit{so} \quad \int_{-2}^0 \frac{1}{x^4}\ dx = \lim_{t\to 0^-}\int_{-2}^t \frac{1}{x^4}\ dx = \lim_{t\to 0^-}\left[-\frac{x^{-3}}{3}\Bigg|_{-2}^t\right]$$

$$= \lim_{t\to 0^-}\left[-\frac{1}{3t^3} - \frac{1}{24}\right] = \infty \quad \textit{so divergent}$$

MATH 151 **Section 2 Worksheet (A)** **Exam 2**

1.

$$\lim_{\theta \to \frac{\pi}{2}} \frac{1 - sin\ \theta}{csc\ \theta}$$

$$= \frac{0}{1} = 0 \quad LH\ does\ not\ apply!$$

2.

$$\lim_{t \to 0} \frac{e^{3t} - 1}{t}$$

$$\stackrel{LH}{=} \lim_{t \to 0} \frac{3e^{3t}}{1} = 3$$

3.

$$\lim_{x \to \infty} \frac{ln\ x}{x}$$

$$\stackrel{LH}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{x} = 0$$

4.

$$\lim_{x \to 0} \frac{tan\ px}{tan\ qx}$$

$$\stackrel{LH}{=} \lim_{x \to 0} \frac{p\ sec^2\ px}{q\ sec^2\ qx} = \frac{p}{q}$$

MATH 151 **Section 2 Worksheet (B)** **Exam 2**

1.

$$\lim_{x\to 0} \quad (cos\ 3x)^{\frac{5}{x}}$$

$ln\ y = \frac{5}{x}\ ln\ cos\ 3x = \lim_{x\to 0}\ \frac{5\ ln\ cos\ 3x}{x} \overset{LH}{=} \lim_{x\to 0}\ \frac{5(-\frac{3sin\ 3x}{cos\ 3x})}{1} = \lim_{x\to 0}(cos\ 3x)^{\frac{5}{x}} = e^0 = 1$

2.

$$\lim_{x\to -1} \frac{x^2-1}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)(x-1)}{(x+1)} = -2$$

3.

$$\lim_{x\to 0} \frac{e^x-1-x}{x^2}$$

$$\overset{LH}{=} \lim_{x\to 0} \frac{e^x-1}{2x} \overset{LH}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$$

4.

$$\lim_{x\to\infty} \left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x$$

$Let\ y = \left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x\ \ so\ \ ln\ y = x\ ln\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x\ so\ \lim_{x\to\infty}\ \frac{ln\left(1+\frac{3}{x}+\frac{5}{x^2}\right)}{\frac{1}{x}} \overset{LH}{=} \lim_{x\to\infty}\ \frac{\frac{-\frac{3}{x^2}-\frac{10}{x^3}}{1+\frac{3}{x}+\frac{5}{x^2}}}{-\frac{1}{x^2}} = \lim_{x\to\infty}\ \frac{3+\frac{10}{x}}{1+\frac{3}{x}+\frac{5}{x^2}} = 3\ \ so\ the\ limit\ is\ e^3$

5.

$$\lim_{x\to\infty} \quad (e^x+x)^{\frac{1}{x}}$$

$Let\ \ y = (e^x+x)^{\frac{1}{x}}\ \ so\ \ ln\ y = \frac{1}{x}\ ln\ (e^x+x)\ \lim_{x\to\infty}\ \frac{ln(e^x+x)}{x} \overset{LH}{=} \lim_{x\to\infty}\ \frac{e^x+1}{e^x+x} \overset{LH}{=} \lim_{x\to\infty}\ \frac{e^x}{e^x+1} \overset{LH}{=} \lim_{x\to\infty}\ \frac{e^x}{e^x} = 1\ \ so\ the\ limit\ is\ e^1 = e$

6.

$$\lim_{x\to\infty} \quad x\ tan\left(\frac{1}{x}\right)$$

$$\lim_{x\to\infty} \frac{tan\frac{1}{x}}{\frac{1}{x}} \overset{LH}{=} \lim_{x\to\infty} \frac{sec^2(\frac{1}{x})(-\frac{1}{x^2})}{(-\frac{1}{x^2})} = \lim_{x\to\infty} \quad sec^2\left(\frac{1}{x}\right) = 1$$

1. Determine whether the sequences converge or diverge. If it converges, find the limit.

   (a)

$$a_n = \frac{2^n}{3^{n+1}}$$

$$\lim_{n\to\infty} \frac{2^n}{3^{n+1}} = \lim_{n\to\infty} \frac{1}{3}\left(\frac{2}{3}\right)^n = \frac{1}{3}(0) = 0 \ \ \textit{thus Converges}$$

   (b)

$$a_n = \frac{(-1)^{n-1}n}{n^2+1}$$

$$\textit{Since} \ \ 0 \leq \mid a_n \mid = \frac{1}{n+\frac{1}{n}} \leq \frac{1}{n}$$

$$\textit{as} \ \ n \to \infty \ \ , \frac{1}{n} \to 0 \ \ \textit{thus by the Squeeze Theorem} \ \ \frac{(-1)^{n-1}\ n}{n^2+1} \ \ \textit{Converges}$$