MATH 151 HW Problems Key Exam 6

Homework Key for Section 1

1. Solve the following:

   (a) $\displaystyle\int arctan\ 4t\, dt$

   $$t\ arctan\ 4t - \frac{1}{8}\ ln(1+16t^2) + C$$

   (b) $\displaystyle\int_0^{\pi} t\ sin\ 3t\, dt$

   $$\frac{\pi}{3}$$

   (c) $\displaystyle\int_0^{\pi/2} sin^2\ x\ cos^2\ x\, dx$

   $$\frac{\pi}{16}$$

2. Find the radius of convergence, R, and the interval of convergence, I, for the following:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

   $$1 \quad , \quad [-1, 1)$$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\ \frac{x^n}{n^3}$

   $$1 \quad , \quad [-1, 1]$$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   $$\infty \quad , \quad (-\infty, \infty)$$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\ \frac{n^2 x^n}{2^n}$

   $$2 \quad , \quad (-2, 2)$$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$

   $$\frac{1}{2} \quad , \quad \left(-\frac{1}{2}, \frac{1}{2}\right]$$

Homework Key for Section 2

1. Solve the following:

   (a) $\displaystyle\int_0^{\pi/3} tan^5\ x\ sec^4\ x\,dx$

$$\frac{117}{8}$$

   (b) $\displaystyle\int \frac{tan^3\ \theta}{cos^4\ \theta}\,d\theta$

$$\frac{tan^6\ \theta}{6}+\frac{tan^4\ \theta}{4}+C$$

   (c) $\displaystyle\int_0^1 x\sqrt{x^2+4}\,dx$

$$\frac{5\sqrt{5}}{3}-\frac{8}{3}$$

   (d) $\displaystyle\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

$$\frac{3}{2}\ ln\ \mid 2x+1\mid -ln\ \mid x-2\mid -\frac{2}{x-2}+C$$

2. Find a power series representation for the following:

   (a)

$$f(x)=\frac{1}{1+x}$$

$$\sum_{n=0}^{\infty}(-1)^n x^n\ \ ,\ \ (-1,1)$$

   (b)

$$f(x)=\frac{x}{9+x^2}$$

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^{n+1}}\ \ ,\ \ (-3,3)$$

3. Find a power series representation for $f(x)=ln(5-x)$ and R.

$$ln\ 5-\sum_{n=1}^{\infty}\frac{x^n}{n5^n}\ \ ,\ \ R=5$$

4. Evaluate $\displaystyle\int \frac{t}{1-t^8}\,dt$ as a power series and find R.

$$C+\sum_{n=0}^{\infty}\frac{t^{8n+2}}{8n+2}\ \ ,\ \ R=1$$

5. Use a power series to approximate $\displaystyle\int_0^{0.2}\frac{1}{1+x^5}\,dx$ to six decimal places

   0.199989

Homework Key for Section 3

1. Find the Taylor Polynomial, $T_n(x)$, for the following functions at the given value a.

   (a) $f(x) = \frac{1}{x}$, $a = 2$

   $$\frac{1}{2} - \frac{(x-2)}{4} + \frac{(x-2)^2}{8} - \frac{(x-2)^3}{16}$$

   (b) $f(x) = cos\ x$, $a = \pi/2$

   $$-\left(x - \frac{\pi}{2}\right) + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$$

2. Approximate $f(x) = \sqrt{x}$ by a Taylor polynomial of degree 2 at $a = 4$.

   $$2 + \frac{(x-4)}{4} - \frac{(x-4)^2}{64}$$

3. Find the Maclaurin series as well as R for the following:

   (a) $f(x) = (1-x)^{-2}$

   $$\sum_{n=0}^{\infty} (n+1)x^n \quad , \quad R = 1$$

   (b) $f(x) = e^{5x}$

   $$\sum_{n=0}^{\infty} \frac{5^n}{n!}x^n \quad , \quad R = \infty$$

4. Find the Taylor series representation for $f(x) = e^x$ centered at $a = 3$.

   $$\sum_{n=0}^{\infty} \frac{e^3}{n!}(x-3)^n$$

5. Evaluate $\int x\ cos(x^3)\, dx$ as an infinite series.

   $$C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{6n+2(2n)!}$$