MATH 151 HW Problems Key Exam 5

Homework Key for Section 1

1. Sketch $x = 1 + \sqrt{t}\ ,\quad y = t^2 - 4t\ ,\quad 0 \leq t \leq 5$
2. Sketch $x = t^2 - 2\ ,\quad y = 5 - 2t\ ,\quad -3 \leq t \leq 4$ and eliminate the parameter to find a Cartesian equation.
$$x = \frac{1}{4}(5 - y)^2 - 2$$
3. Eliminate the parameter and sketch $x = sin\ \theta\ ,\quad y = cos\ \theta\ ,\quad 0 \leq \theta \leq \pi$
$$x^2 + y^2 = 1\ ,\quad x \geq 0$$
4. Describe the motion of $x = 5sin\ t\ ,\quad y = 2cos\ t\ ,\quad -\pi \leq t \leq 5\pi$
$$\text{moves three times clockwise around the ellipse } \frac{x^2}{25} + \frac{y^2}{4} = 1$$

Homework Key for Section 2

1. Find an equation of the tangent line to $x = t^4 + 1$, $y = t^3 + 4$ at $t = -1$

$$y - 3 = -3/4(x - 2)$$

2. Find an equation of the tangent line to $x = e^{\sqrt{t}}$, $y = t - ln\ t^2$ at $t = 1$

$$y = -\frac{2x}{e} + 3$$

3. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the following as well as when the curves are CU.

   (a) $x = 4 + t^2$, $y = t^2 + t^3$

$$1 + \frac{3}{2}t \ , \ \frac{3}{4t} \ , \ t > 0$$

   (b) $x = t - e^t$, $y = t + e^{-t}$

$$-e^{-t} \ , \ \frac{e^{-t}}{1 - e^t} \ , \ t < 0$$

4. SET UP ONLY the integral that represents the length of

$$x = t - t^2 \ , \ y = 4/3t^{3/2} \ , \ 1 \leq t \leq 2$$

$$\int_1^2 \sqrt{1 + 4t^2}\, dt$$

5. Find the exact length of $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$

$$4\sqrt{2} - 2$$

6. Find the exact length of $x = e^t cos\ t$, $y = e^t sin\ t$, $0 \leq t \leq \pi$

$$\sqrt{2}(e^{\pi} - 1)$$

7. Find the surface area by rotating $x = acos^3\ \theta$, $y = asin^3\ \theta$, $0 \leq \theta \leq \pi/2$ about the x-axis.

$$\frac{6}{5}\pi a^2$$

Homework Key for Section 3

1. Sketch the following region: $5\pi/3 \leq \theta \leq 7\pi/3$ for $2 < r < 3$

2. Identify by finding a Cartesian equation for $r = 2$

circle centered at the origin of radius 2

3. Identify by finding a Cartesian equation for $r = 3\ sin\ \theta$

circle centered at (0,3/2) with radius 3/2

4. Find a polar equation for the following:

   (a) $x = 3$

   $$r = 3\ sec\ \theta$$

   (b) $x^2 + y^2 = 2cx$

   $$r = 2c\ cos\ \theta$$

5. Sketch the following:

   (a) $r = sin\ \theta$

   (b) $r = 2(1 - sin\ \theta), \quad \theta \geq 0$

   (c) $r = \theta, \quad \theta \geq 0$

6. Find the slope of the tangent line to $r = 2\ sin\ \theta$ at $\theta = \pi/6$

$$\sqrt{3}$$

7. Find the slope of the tangent line to $r = 1/\theta$ at $\theta = \pi$

$$-\pi$$

8. Find the slope of the tangent line to $r = cos\ 2\theta$ at $\theta = \pi/4$

$$1$$

9. Find the points on $r = 3\ cos\ \theta$ where the tangent line is horizontal or vertical.

horizontal at $(3/\sqrt{2}, \pi/4), (-3/\sqrt{2}, 3\pi/4)$ vertical at $(3, 0), (0, \pi/2)$

Homework Key for Section 4

1. Find the area of the region bounded by $r = sin\ \theta$ on $\pi/3 \leq \theta \leq 2\pi/3$

$$\frac{\pi}{12} + \frac{\sqrt{3}}{8}$$

2. Sketch $r = 3\ cos\ \theta$ and find the area it encloses.

$$\frac{9\pi}{4}$$

3. Find the area enclosed by ONE loop of $r = sin\ 2\theta$

$$\frac{\pi}{8}$$

4. Find the area that lies inside $r = 3\ cos\ \theta$ and outside $r = 1 + cos\ \theta$

$$\pi$$

5. Find the area that lies in both $r = \sqrt{3}\ cos\ \theta$ and $r = sin\ \theta$

$$\frac{5\pi}{24} - \frac{\sqrt{3}}{4}$$

6. Find the exact length of the polar curve $r = 3\ sin\ \theta$ from $0 \leq \theta \leq \pi/3$

$$\pi$$