# MATH 150 Review Key Exam 6

1. Evaluate the integral by interpreting it in terms of areas.

   (a) $\int_{-1}^{4} 4-2x \; dx$ triangle $\ldots = 13$

   (b) $\int_{-2}^{2} \sqrt{4-x^2} \; dx$ one half of a circle with radius 2 $\ldots = 2\pi$

   (c) $\int_{-1}^{2} |\, x \,| \; dx$ two triangles $\ldots = \frac{5}{2}$

2. Evaluate the integral USING THE DEFINITION.

   (a) $\int_{1}^{4} x^2+2x-5 \; dx$ since $\Delta x = \frac{3}{n}$ and $x_i = 1+\frac{3i}{n}$ the definition yields

   $$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left[ \left(1+\frac{3i}{n}\right)^2 + 2\left(1+\frac{3i}{n}\right) - 5 \right] \frac{3}{n} = \ldots = 21$$

   (b) $\int_{0}^{5} 1+2x^3 \; dx$ since $\Delta x = \frac{5}{n}$ and $x_i = \frac{5i}{n}$ the definition yields

   $$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left[ 1+2\left(\frac{5i}{n}\right)^3 \right] \frac{5}{n} = \ldots = \frac{635}{2}$$

3. $\int_{0}^{1} (1-x^9)\, dx = x - \frac{x^{10}}{10}\Big|_0^1 = \frac{9}{10}$

4. $\int_{0}^{1} (\sqrt[4]{u}+1)^2 \, du \quad \int_{0}^{1} (u^{1/2}+2u^{1/4}+1)\, du = \frac{2}{3}u^{3/2} + \frac{8}{5}u^{5/4} + u\Big|_0^1 = \frac{49}{15}$

5. $\int_{0}^{1} 3+x\sqrt{x} \; dx = \int_{0}^{1} 3+x^{3/2} \; dx \; \ldots = \frac{17}{5}$

6. $\int_{1}^{4} \frac{1}{\sqrt{x}} \; dx = \ldots = 2$

7. $\int sec^2 \, t \; dt = \ldots = \tan x + C$

8. $\int_{-5}^{5} \frac{2}{x^3} \; dx$ The function is discontinuous at $x = 0$ so FTC does NOT apply.

9. $\int_{1}^{2} \frac{4+u^2}{u^3} \; du = \int_{1}^{2} 4u^{-3} \; du + \int_{1}^{2} \frac{1}{u} \; du \; \ldots = \frac{3}{2} + \ln 2$

10. $\int \frac{x}{\sqrt{1-4x^2}}\ dx$

since $u = 1 - 4x^2$ and $du = -8x\ dx$ we have

$\ldots = -\frac{1}{4}\sqrt{1-4x^2} + C$

11. $\int x^2\sqrt{2+x}\ dx$

since $u = 2 + x$ and $du = dx$ then $x = u - 2$ and $x^2 = (u-2)^2$ thus

$\ldots = \frac{2}{7}(2+x)^{7/2} - \frac{8}{5}(2+x)^{5/2} + \frac{8}{3}(2+x)^{3/2} + C$

12. $\int \frac{4}{x(ln\ x)^2}\ dx$

since $u = \ln x$ and $du = \frac{1}{x}\ dx$ we have

$\ldots = -\frac{4}{\ln x} + C$

13. $\int_0^1 \sqrt[3]{1+7x}\ dx$

since $u = 1 + 7x$ and $du = 7\ dx$ we have

$\ldots = \frac{45}{28}$

14. $\int x(2x+5)^8\ dx$

since $u = 2x + 5$ and $du = 2\ dx$ then $x = \frac{u-5}{2}$ and

$\ldots = \frac{(2x+5)^{10}}{40} - \frac{5}{36}(2x+5)^9 + C$

15. $\int e^x\sqrt{1+e^x}\ dx$

since $u = 1 + e^x$ then $du = e^x\ dx$ and

$\ldots = \frac{2}{3}(1+e^x)^{3/2} + C$

16. $\int \frac{e^{\sqrt{t}+1}}{\sqrt{t}}\ dt$

since $u = \sqrt{t} + 1$ then $du = \frac{1}{2\sqrt{t}}\ dt$ and

$\ldots = 2e^{\sqrt{t}+1} + C$

17. $\int 2^{3-4t}\ dt$

since $u = 3 - 4t$ then $du = -4\ dt$ and

$\ldots = -\frac{2^{3-4t}}{4\ \ln 2} + C$

18. $\displaystyle\int_0^1 (1-x)^9\,dx$

since $u = 1-x$ and $du = -dx$ we have

$$-\int_0^1 (1-x)^9\,dx \;=\; -\int_0^1 u^9\,du \;=\; -\frac{u^{10}}{10}\Big|_0^1 \;=\; \frac{1}{10}$$

19. $\displaystyle\int_0^1 y(y^2+1)^5\,dy$

since $u = y^2+1$ and $du = 2y\,dy$ we have

$$\tfrac{1}{2}\int_0^1 2y(y^2+1)^5\,dy \;=\; \frac{1}{2}\int_0^1 u^5\,du \;=\; \frac{1}{2}\frac{u^6}{6}\Big|_0^1 \;=\; \frac{1}{12}(y^2+1)^6\Big|_0^1 \;=\; \frac{21}{4}$$

20. $\displaystyle\int_1^2 \frac{1}{2-3x}\,dx$

since $u = 2-3x$ and $du = -3\,dx$ we have

$$-\tfrac{1}{3}\int_1^2 \frac{-3}{2-3x}\,dx \;=\; -\frac{1}{3}\int_1^2 \frac{1}{u}\,du \;=\; -\frac{1}{3}\ln\,|\,u\,|\,\Big|_1^2 \;=\; -\frac{1}{3}\ln\,|\,2-3x\,|\,\Big|_1^2 \;=\; -\frac{1}{3}\ln\,4$$

21. $\displaystyle\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$

since $u = x^2+4x$ and $du = 2x+4\,dx$ we have

$$\tfrac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+4x}}\,dx \;=\; \frac{1}{2}\int u^{-1/2}\,du \;=\; \frac{1}{2}\,2u^{1/2}+C \;=\; \sqrt{x^2+4x}+C$$

22. Find the area of the region bounded by the given curves:

(a) $y = x^2-x-6, \quad y = 0$

$$\mathbf{A}= \int_{-2}^3 (0-(x^2-x-6))\,dx \;=\; \frac{125}{6}$$

(b) $y = 20-x^2, \quad y = x^2-12$

$$\mathbf{A}= \int_{-4}^4 [(20-x^2)-(x^2-12)]\,dx \;=\; \frac{512}{3}$$

(c) $x+y = 0, \quad x = y^2+3y$

$$\mathbf{A}= \int_{-4}^0 [(-y)-(y^2+3y)]\,dy \;=\; \frac{32}{3}$$