MATH 151 Exam 2 Review Key Spring

Evaluate the following integrals:

1. $\displaystyle\int_0^3 \frac{1}{\sqrt{x}}\ dx$

$$= \lim_{t\to 0^+} \int_t^3 \frac{1}{\sqrt{x}}\ dx \ = \ 2\sqrt{3} \implies \text{Convergent}$$

2. $\displaystyle\int_{2\pi}^{\infty} sin\ \theta\ \ d\theta$

$$= \lim_{t\to\infty} \int_{2\pi}^{t} sin\ \theta\ \ d\theta \ = \ \lim_{t\to\infty}\ (-cos\ t + 1) \implies DNE \text{ so Divergent}$$

3. $\displaystyle\int_1^{\infty} \frac{ln\ x}{x}\ dx$

$$= \lim_{t\to\infty} \left.\frac{(ln\ x)^2}{2}\right|_1^t \ = \ \infty \text{ so Divergent}$$

4. Use the Comparison Theorem to determine convergence or divergence.

   (a) $\displaystyle\int_0^1 \frac{e^{-x}}{\sqrt{x}}\ dx$

$$\frac{e^{-x}}{\sqrt{x}} \leq \frac{1}{\sqrt{x}}$$

$$\int_0^1 \frac{1}{\sqrt{x}}\ dx \ = \ \lim_{t\to 0^+} \int_t^1 \frac{1}{\sqrt{x}}\ dx \ = \ 2 \implies \text{Convergent}$$

$$\text{therefore } \int_0^1 \frac{e^{-x}}{\sqrt{x}}\ dx \ \text{ also converges}$$

   (b) $\displaystyle\int_0^{\infty} \frac{x}{x^3+1}\ dx$

$$\frac{x}{x^3+1} < \frac{x}{x^3} \ = \ \frac{1}{x^2}$$

$$\int_0^{\infty} \frac{x}{x^3+1}\ dx \ = \ \int_0^1 \frac{x}{x^3+1}\ dx + \int_1^{\infty} \frac{x}{x^3+1}\ dx \implies \text{Finite area + Convergent}$$

$$\text{therefore } \int_0^{\infty} \frac{x}{x^3+1}\ dx \ \text{ also converges}$$

(c) $\displaystyle\int_0^{\pi/2} \frac{dx}{x\ sin\ x}$

$$\frac{1}{x\ sin\ x} \geq \frac{1}{x}$$

$$\int_0^{\pi/2} \frac{dx}{x} \ = \lim_{t\to 0^+} \ ln\ x\Big|_t^{\pi/2} \ = \ \infty \ \Longrightarrow \ \int_0^{\pi/2} \frac{dx}{x\ sin\ x} \text{ is divergent}$$

5. Show whether the integral is convergent or divergent. Evaluate it if it is convergent.

   (a) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\ dx$

   see HW key for section 1 problem 1e

6. Find $\lim_{x\to\infty} \ x\ sin(\pi/x)$

$$\text{this is type } \infty \cdot 0 \ \text{ so } \ = \ \lim_{x\to\infty} \frac{sin(\pi/x)}{\frac{1}{x}} \ = \ \text{type } \frac{0}{0}$$

$$\overset{LH}{=} \ \lim_{x\to\infty} \frac{cos(\pi/x)\left(\frac{-1}{x^2}\right)(\pi)}{\frac{-1}{x^2}} \ = \ \pi$$

7. Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

$$a_n = \frac{ln\ n}{\sqrt{n}}$$

$$\text{look at the associated function } \lim_{x\to\infty} \frac{ln\ x}{\sqrt{x}} \text{ which is type } \frac{\infty}{\infty}$$

$$\overset{LH}{=} \ \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} \ = \ \lim_{x\to\infty} \frac{2}{\sqrt{x}} \ = \ 0$$

8. Determine whether the sequence converges or diverges. If it converges, find its limit.

$$a_n \ = \ \frac{2^n}{3^{n+1}}$$

$$\lim_{n\to\infty} \frac{2^n}{3^{n+1}} \ = \ \left(\frac{1}{3}\right) \lim_{n\to\infty} \left(\frac{2}{3}\right)^n \ = \ 0 \ \text{ so Convergent}$$