MATH 150 Worksheets Exam 6

Worksheet for Section 1

1. Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using:

   (a) three rectangles and right endpoints.

   $$\text{For 3 rectangles, each has width 1} \quad \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3}$$
   $$R_3 = \Delta x[f(0) + f(1) + f(2)] = 1[1 + 2 + 5] = 8$$

   (b) six rectangles and right endpoints.

   $$\text{For 6 rectangles, each has width 1/2} \quad \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{6}$$
   $$R_6 = \Delta x[f(-1/2)+f(0)+f(1/2)+f(1)+f(3/2)+f(2)] = \frac{1}{2}\left[\frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4} + 5\right] = \frac{55}{8}$$

   (c) three rectangles and left endpoints.

   $$\text{For 3 rectangles, each has width 1} \quad \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3}$$
   $$L_3 = \Delta x[f(-1) + f(0) + f(1)] = 1[2 + 1 + 2] = 5$$

   (d) six rectangles and left endpoints.

   $$\text{For 6 rectangles, each has width 1/2} \quad \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{6}$$
   $$L_6 = \Delta x[f(-1)+f(-1/2)+f(0)+f(1/2)+f(1)+f(3/2)] = \frac{1}{2}\left[2 + \frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4}\right] = \frac{43}{8}$$

   (e) three rectangles and midpoints.

   $$\text{For 3 rectangles, each has width 1} \quad \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3}$$
   $$M_3 = \Delta x[f(-1/2) + f(1/2) + f(3/2)] = 1\left[\frac{5}{4} + \frac{5}{4} + \frac{13}{4}\right] = \frac{23}{4}$$

   (f) six rectangles and midpoints.

   $$\text{For 6 rectangles, each has width 1/2} \quad \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{6}$$
   $$M_6 = \Delta x[f(-3/4) + f(-1/4) + f(1/4) + f(3/4) + f(5/4) + f(7/4)]$$
   $$= \frac{1}{2}\left[\frac{25}{16} + \frac{17}{16} + \frac{17}{16} + \frac{25}{16} + \frac{41}{16} + \frac{65}{16}\right] = \frac{95}{16}$$

Worksheet for Section 2

1. For the following two questions, let $f(x) = x^3 - 6x$ and use endpoints $a = 0$ and $b = 3$.

   (a) Evaluate R_6

   For 6 rectangles, each has width 1/2 $\Delta x = \dfrac{b-a}{n} = \dfrac{3-0}{6}$

   $$R_6 = \Delta x[f(1/2)+f(1)+\ldots+f(3)] = \frac{1}{2}[-2.875 - 5 - 5.625 - 4 + .625 + 9] = -3.9375$$

   (b) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

   $$\Delta x = \frac{b-a}{n} = \frac{3}{n} \quad , \quad x_i = a + i\Delta x = 0 + \frac{3i}{n} = \frac{3i}{n}$$

   $$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right]\frac{3}{n}$$

   $$= \lim_{n\to\infty} \frac{81}{n^4} \sum_{i=1}^{n} i^3 - \frac{54}{n^2} \sum_{i=1}^{n} i = \lim_{n\to\infty} \left[\frac{81}{n^4}\left(\frac{n(n+1)}{2}\right)^2 - \frac{54}{n^2}\left(\frac{n(n+1)}{2}\right)\right]$$

   $$= \frac{81}{4} - \frac{54}{2} = -6.75$$

Worksheet for Section 3

1. Approximate

$$\int_1^2 \frac{ln\ x}{1+x}\ dx$$

using:

(a) M_{10}

$$M_{10} = \frac{1}{10}\left[f(1.05) + f(1.15) + \ldots + f(1.95)\right] \approx .147391$$

(b) T_{10}

$$T_{10} = \frac{1}{10 \cdot 2}\left[f(1) + 2f(1.1) + 2f(1.2) + \ldots + f(2)\right] \approx .146879$$

(c) S_{10}

$$S_{10} = \frac{1}{10 \cdot 3}\left[f(1) + 4f(1.1) + 2f(1.2) + \ldots + 4f(1.9) + f(2)\right] \approx .147219$$

Worksheet for Section 4

1. Use the FTC to evaluate the following integrals:

   (a) $\int_{-2}^{5} 6\,dx$

$$\int_{-2}^{5} 6\,dx \;=\; 6x\,\Big|_{-2}^{5} \;=\; 42$$

   (b) $\int_{0}^{4} (1+3y-y^2)\,dy$

$$\int_{0}^{4} (1+3y-y^2)\,dy \;=\; y+\frac{3}{2}y^2-\frac{y^3}{3}\,\Big|_{0}^{5} \;=\; \frac{20}{3}$$

   (c) $\int_{\pi}^{2\pi} cos\;\theta\,d\theta$

$$\int_{\pi}^{2\pi} cos\;\theta\,d\theta \;=\; sin\;\theta\,\Big|_{\pi}^{2\pi} \;=\; 0$$

   (d) $\int_{0}^{1} \frac{4}{t^2+1}\,dt$

$$=\;4\int_{0}^{1} \frac{1}{t^2+1}\,dt \;=\; 4\;arctan\;t\,\Big|_{0}^{1} \;=\; 4\left(\frac{\pi}{4}-0\right)=\pi$$

Worksheet for Section 5

1. Find the general indefinite integral:

   (a) $\int (x^2 + 1 + \frac{1}{x^2+1})\, dx$

   $$= \frac{x^3}{3} + x + arctan\ x + C$$

   (b) $\int (3e^u + sec^2\ u)\, du$

   $$= 3e^u + tan\ u + C$$

2. Evaluate:

   (a) $\int_0^5 (2e^x + 4cos\ x)\, dx$

   $$= 2e^x + 4sin\ x\Big|_0^5 = 2e^5 + 4sin\ 5 - [2e^0 + 4sin\ 0]$$

   $$= 2e^5 + 4sin\ 5 - 2$$

   (b) $\int_1^4 \sqrt{\frac{5}{x}}\, dx$

   $$= \sqrt{5} \int_1^4 x^{-1/2}\, dx = 2\sqrt{5}\sqrt{x}\Big|_1^4 = 2\sqrt{5}(\sqrt{4} - \sqrt{1}) = 2\sqrt{5}$$

Worksheet for Section 6

1. Find the general indefinite integral:

   (a) $\int 2x(x^2+3)^4\,dx$

$$u = x^2 + 3 \;\text{ so the derivative is }\; du = 2x\;dx$$

$$\int u^4\;du \;=\; \frac{u^5}{5} + C \;=\; \frac{(x^2+3)^5}{5} + C$$

   (b) $\int \frac{sin\;x}{1+cos^2\;x}\,dx$

$$u = cos\;x \;\text{ so the derivative is }\; du = -sin\;x\;dx$$

$$-\int \frac{1}{1+u^2}\;du \;=\; -\,arctan\;u + C \;=\; -\,arctan(cos\;x) + C$$

2. Evaluate:

   (a) $\int_1^2 x(\sqrt{x-1})\,dx$

$$u = x - 1 \;\text{ so the derivative is }\; du = dx \;\text{ and notice that }\; x = u + 1$$

$$\int_1^2 (u+1)\sqrt{u}\;du \;=\; \int_1^2 (u^{3/2} + u^{1/2})\;du \;=\; \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big|_1^2$$

$$= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}\Big|_1^2 \;=\; \left(\frac{2}{5} + \frac{2}{3}\right) - 0 \;=\; \frac{16}{15}$$

Worksheet for Section 7

1. Sketch the region enclosed by $y = sin\ x$, $y = e^x$, $x = 0$, $x = \pi/2$. Draw a typical rectangle and find the area of the region.

$$A = \int_0^{\pi/2} (e^x - sin\ x)\ dx \ = \ e^{\pi/2} - 2$$

2. Sketch the region enclosed by $y = x^3 - x$, $y = 3x$. Draw a typical rectangle and find the area of the region.

$$A = \int_{-2}^{2} \Big| 3x - (x^3 - x) \Big|\ dx \ \text{ or by symmetry}$$

$$2 \int_0^2 [3x - (x^3 - x)]\ dx \ = \ 8$$

3. Sketch the region enclosed by $4x + y^2 = 12$, $y = x$. Draw a typical rectangle and find the area of the region.

$$A = \int_{-6}^{2} (x_R - x_L)\ dy \ = \ \int_{-6}^{2} \left[\left(-\frac{y^2}{4} + 3 \right) - (y) \right]\ dy \ = \ \frac{64}{3}$$