MATH 150 Worksheet Keys Exam 3

Worksheet for Section 1

1. Find the derivatives of the following functions:

   (a)

$$f(x) = \sqrt{30}$$

$$f'(x) = 0$$

   (b)

$$f(x) = 5e^x + 3$$

$$f'(x) = 5e^x$$

   (c)

$$f(x) = \frac{\sqrt{10}}{x^7}$$

$$f'(x) = -7\sqrt{10}x^{-8}$$

   (d)

$$f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

2. Find the equation of the tangent line to $y = x^4 + 2e^x$ at $(0, 2)$

$f'(x) = 4x^3 + 2e^x \quad \Longrightarrow \quad$ the slope at $(0, 2)$ is $f'(0) = 2$

so the equation is $y - 2 = 2(x - 0)$ or $y = 2x + 2$

Worksheet for Section 2

1. Find the derivatives of the following functions:

   (a)

$$f(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$$

$$f'(x) = \frac{x}{\sqrt{x}} - \frac{3x\sqrt{x}}{\sqrt{x}} = x^{1/2} - 3x$$

$$f'(x) = \frac{1}{2}x^{-1/2} - 3$$

   (b)

$$g(x) = \sqrt{x}e^x$$

$$g'(x) = \sqrt{x}\left[\frac{d}{dx}(e^x)\right] + e^x\left[\frac{d}{dx}(\sqrt{x})\right]$$

$$= \sqrt{x}e^x + e^x\frac{1}{2}x^{-1/2}$$

   (c)

$$y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$$

$$y' = \frac{(\sqrt{x}+1)\frac{d}{dx}(\sqrt{x}-1) - \left[(\sqrt{x}-1)\frac{d}{dx}(\sqrt{x}+1)\right]}{(\sqrt{x}+1)^2} = \frac{(\sqrt{x}+1)\frac{1}{2}x^{-1/2} - \left[(\sqrt{x}-1)\frac{1}{2}x^{-1/2}\right]}{(\sqrt{x}+1)^2}$$

$$= \frac{x^{-1/2}}{(\sqrt{x}+1)^2}$$

2. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$ find $(fg)'(3)$

$$(fg)'(3) = f(3)g'(3) + g(3)f'(3) = 4(5) + 2(-6) = 8$$

Worksheet for Section 3

1. Find the derivative of $f(x) = x\ sin\ x$

$$f'(x) = x\frac{d}{dx}(sin\ x) + sin\ x\frac{d}{dx}(x) \ = \ x\ cos\ x + sin\ x$$

2. Find an equation of the tangent line to $y = e^x\ cos\ x$ at $(0, 1)$

$$y' = e^x\frac{d}{dx}(cos\ x) + cos\ x\frac{d}{dx}(e^x) \ = \ -e^x sin\ x + e^x cos\ x \ = \ e^x(cos\ x - sin\ x)$$

$$\text{at the point } (0,1) \quad \Longrightarrow \quad y' = e^0(cos\ 0 - sin\ 0) = 1 \quad \Longrightarrow \quad y - 1 = x \ \text{ or } \ y = x + 1$$

3. Evaluate

$$\lim_{t\to 0} \frac{sin^2\ 3t}{t^2}$$

$$= \ \lim_{t\to 0} \left(\frac{sin\ 3t}{t}\right)\left(\frac{sin\ 3t}{t}\right) \ = \ \left(\lim_{t\to 0} \frac{3sin\ 3t}{3t}\right)\left(\lim_{t\to 0} \frac{3sin\ 3t}{3t}\right)$$

$$= \ 3^2 \left(\lim_{t\to 0} \frac{sin\ 3t}{3t}\right)^2 \ = \ 3^2 \ = \ 9$$

Worksheet for Section 4

1. Find the derivative of the following functions:

(a) $F(x) = \sqrt{1 + 2x + x^3}$

$$F'(x) = \frac{1}{2}(1 + 2x + x^3)^{-1/2}(2 + 3x^2)$$

(b) $g(t) = \frac{1}{(t^4+1)^3}$

$$g'(t) = -3(t^4 + 1)^{-4}(4t^3)$$

(c) $y = cos(x^3)$

$$y' = -sin(x^3)(3x^2)$$

(d) $y = e^{-\pi x}$

$$y' = e^{-\pi x}(-\pi)$$

(e) $y = xe^{-x^2}$

$$y' = x\frac{d}{dx}(e^{-x^2}) + e^{-x^2}\frac{d}{dx}(x) \;=\; xe^{-x^2}(-2x) + e^{-x^2} \;=\; e^{-x^2}(1 - 2x^2)$$

(f) $y = e^{xcosx}$

$$y' = e^{xcosx}\frac{d}{dx}(x\ cos\ x) \;=\; e^{xcosx}(x(-sin\ x) + cos\ x)$$

(g) $F(z) = \sqrt{\frac{z-1}{z+1}}$

$$F'(z) = \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2}\frac{d}{dz}\left(\frac{z-1}{z+1}\right) \;=\; \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2}\left(\frac{(z+1)(1)-(z-1)(1)}{(z+1)^2}\right)$$

$$= \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2}\left(\frac{2}{(z+1)^2}\right)$$

Worksheet for Section 5

1. Find dy/dx by implicit differentiation:

   (a) $\sqrt{x+y} = 1 + x^2y^2$

$$\frac{1}{2}(x+y)^{-1/2}\left(1+\frac{dy}{dx}\right) = x^2 2y\frac{dy}{dx} + y^2 2x \iff \frac{1}{2\sqrt{x+y}} + \frac{dy/dx}{2\sqrt{x+y}} = 2x^2y\frac{dy}{dx} + 2xy^2$$

$$\frac{dy}{dx}\left[\frac{1}{2\sqrt{x+y}} - 2x^2y\right] = 2xy^2 - \frac{1}{2\sqrt{x+y}} \iff \frac{dy}{dx} = \frac{2xy^2 - 1/(2\sqrt{x+y})}{1/(2\sqrt{x+y}) - 2x^2y}$$

$$= \frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}}$$

   (b) $y^5 + x^2y^3 = 1 + ye^{x^2}$

$$5y^4y' + x^2 3y^2y' + y^3 2x = ye^{x^2}2x + e^{x^2}y' \iff y'(5y^4 + 3x^2y^2 - e^{x^2}) = 2xye^{x^2} - 2xy^3$$

$$\implies y' = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}$$

2. Differentiate $y = cos^{-1}(e^{2x})$

$$y' = -\frac{1}{\sqrt{1-(e^{2x})^2}}\left(\frac{d}{dx}(e^{2x})\right) = -\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$$