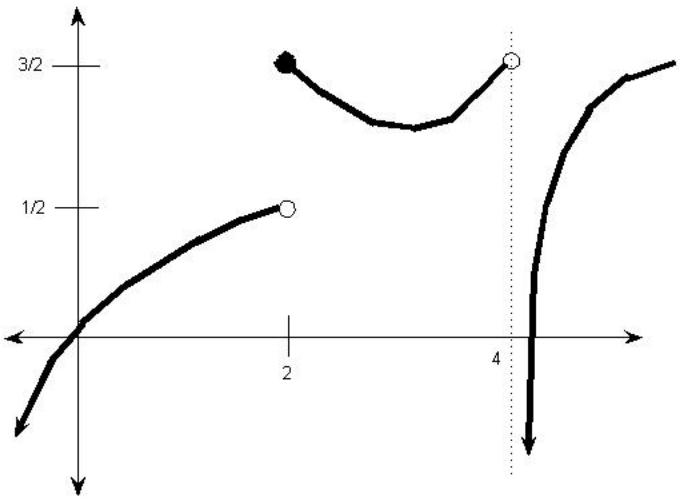
MATH 150 Worksheet Keys Exam 2

Worksheet for Section 1



1. For the given function, state the value of each quantity, if it exists. If it does not exist, explain why.

   (a) $\lim_{x\to 2^-} g(x)$ $= \frac{1}{2}$

   (b) $\lim_{x\to 2^+} g(x)$ $= \frac{3}{2}$

   (c) $\lim_{x\to 2} g(x)$ DNE

   (d) $g(2)$ $= \frac{3}{2}$

   (e) $\lim_{x\to 4^-} g(x)$ $= \frac{3}{2}$

   (f) $\lim_{x\to 4^+} g(x)$ DNE $(-\infty)$

2. Determine the infinite limit $\lim_{x\to 5^+} ln(x-5)$

   inside the parentheses are small positive numbers $\Longrightarrow$ $-\infty$

3. Use a table of values to estimate the value of $\lim_{x\to 0} \frac{9^x - 5^x}{x}$

   you should get $\approx$ .59

Worksheet for Section 2

1. Determine the limit if it exists $\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4}$

$$\lim_{x\to -4} \frac{(x+4)(x+1)}{(x+4)(x-1)} = \lim_{x\to -4} \frac{x+1}{x-1} = \frac{3}{5}$$

2. Find $\lim_{x\to 2} \frac{|x-2|}{x-2}$ if it exists. If it does not exist, explain why.

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1 \quad for\ x > 2$$

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{-(x-2)}{x-2} = -1 \quad for\ x < 2$$

$$\Longrightarrow \quad DNE$$

3. Show by means of an example that $\lim_{x\to a} [f(x)g(x)]$ may exist even though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists.

answers may vary but look at

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \quad and \quad g(x) = \begin{cases} 1, & x < 0 \\ 0, & x \geq 0 \end{cases}$$

$$\lim_{x\to 0} f(x) \quad DNE \quad and \quad \lim_{x\to 0} g(x) \quad DNE \quad but \quad \lim_{x\to 0} f(x)g(x)] = 0$$

Worksheet for Section 3

1. Use a graph of $f(x) = 1/x$ to find a number δ such that $|\ \frac{1}{x} - 0.5\ | < 0.2$ whenever $|\ x - 2\ | < \delta$.

$$\text{on the left side we need } |x-2| < \left|\frac{10}{7} - 2\right| = \frac{4}{7}$$

$$\text{on the right side we need } |x-2| < \left|\frac{10}{3} - 2\right| = \frac{4}{3}$$

$$\text{since we need the smaller number } \delta = \frac{4}{7} \text{ or anything smaller}$$

2. Prove, using the δ , ϵ definition of a limit, that $\lim_{x \to -2} (\frac{1}{2}x + 3) = 2$

we need $|f(x) - L| < \epsilon$, that is, $\left|\frac{1}{2}x + 3 - 2\right| = \left|\frac{1}{2}x + 1\right| = \left|\frac{1}{2}\right||x+2| < \epsilon$ so choose $\delta = 2\epsilon$

Proof:

$$\text{given } \epsilon > 0, \text{ choose } \delta = 2\epsilon \text{ if } |x+2| < \delta \implies$$

$$\left|\frac{1}{2}x + 3 - 2\right| = \left|\frac{1}{2}x + 1\right| = \frac{1}{2}|x+2| < \frac{1}{2}\delta = \frac{1}{2}(2\epsilon = \epsilon)$$

$$\text{thus } |x+2| < \delta \implies |f(x) - L| < \epsilon \qquad \blacksquare$$

Worksheet for Section 4

1. Show that $f(x)$ is continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} x^2, & x < 1 \\ \sqrt{x}, & x \geq 1 \end{cases}$$

$$x^2 \text{ is a polynomial so } \Longrightarrow f \text{ is continuous on } (-\infty, 1)$$
$$\sqrt{x} \text{ is a root function so } \Longrightarrow f \text{ is continuous on } (1, \infty)$$
$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} x^2 = 1 \text{ and } \lim_{x\to1^+} f(x) = \lim_{x\to1^+} \sqrt{x} = 1$$
$$\text{since } f(1) = \sqrt{1} = 1 \quad \Longrightarrow \quad f \text{ is continous on } (-\infty, \infty)$$

2. Why is the following function discontinuous at $a = 1$?

$$g(x) = \begin{cases} \frac{x^2-x}{x^2-1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$$

$$\lim_{x\to1} g(x) = \lim_{x\to1} \frac{x^2 - x}{x^2 - 1} = 1 = \lim_{x\to1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x\to1} \frac{x}{x+1} = \frac{1}{2}$$
$$\text{since } f(1) = 1 \quad \Longrightarrow \quad f \text{ is discontinous at } x = 1$$

3. Is there a number that is exactly one more than its cube?

$$\text{if so then } x^3 + 1 = x \Longrightarrow x^3 - x + 1 = 0 \text{ is continuous on } (-\infty, 1)$$

$$\text{note that } f(-2) = -5 \;\; \textit{and} \;\; f(-1) = 1$$
$$f \text{ is a polynomial so continuous everywhere.}$$
$$\text{By the IVT } \exists \text{ a } c \text{ between } -2 \text{ and } -1 \text{ such that } f(c) = 0$$

Worksheet for Section 5

1. Sketch a graph of a function that satisfies the following conditions:

   (a) $\lim_{x \to -2} f(x) = \infty$

   (b) $\lim_{x \to -\infty} f(x) = 3$

   (c) $\lim_{x \to \infty} f(x) = -3$

answers may vary slightly here

2. Find $\lim_{x \to \infty} \frac{x^3 - 2x + 3}{5 - 2x^2}$

$$\lim_{x \to \infty} \frac{\frac{x^3}{x^2} - \frac{2x}{x^2} + \frac{3}{x^2}}{\frac{5}{x^2} - \frac{2x^2}{x^2}} = \lim_{x \to \infty} \frac{x}{-2} = -\infty$$

3. Find $\lim_{x \to \infty} tan^{-1}(x^2 - x^4)$

$$\lim_{x \to \infty} tan^{-1}(x^2(1 - x^2))$$

note that as $x \to \infty$ $x^2(1 - x^2) = \infty(-\infty) = -\infty \implies -\frac{\pi}{2}$

Worksheet for Section 6

1. If a ball is thrown into the air with a velocity of 40 ft/s, its height, in feet, after t seconds is given by $y = 40t - 16t^2$. Find the average velocity from $t = 1$ to $t = 2$ as well as the velocity when $t = 2$.

$$V_{avg} \; \frac{s(2)-s(1)}{2-1} = -8\text{ft/sec}$$

$$v(2) = \lim_{t\to 2} \frac{s(t)-s(2)}{t-2} = \lim_{t\to 2} \frac{40t-16t^2-16}{t-2} = \lim_{t\to 2} \frac{-8(t-2)(2t-1)}{t-2}$$

$$= -8 \lim_{t\to 2} (2t-1) = -24\text{ft/sec}$$

2. Find $f'(a)$ if

$$f(x) = \frac{1}{\sqrt{x+2}}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{a+h+2}} - \frac{1}{\sqrt{x+2}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{a+2}-\sqrt{a+h+2}}{\sqrt{a+h+2}\sqrt{a+2}}}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sqrt{a+2}-\sqrt{a+h+2}}{h\sqrt{a+h+2}\sqrt{a+2}} \cdot \frac{\sqrt{a+2}+\sqrt{a+h+2}}{\sqrt{a+2}+\sqrt{a+h+2}}\right]$$

$$= \lim_{h\to 0} \frac{-h}{h\sqrt{a+h+2}\sqrt{a+2}(\sqrt{a+2}+\sqrt{a+h+2})} = \frac{-1}{(\sqrt{a+2})^2 2(\sqrt{a+2})} = -\frac{1}{2(a+2)^{3/2}}$$

3. Given that

$$\lim_{h\to 0} \frac{cos(\pi+h)+1}{h}$$

represents $f'(a)$, what is the function f and the number a?

$$f(x) = cos\ x \text{ and } a = \pi \quad \Longrightarrow \quad f'(\pi) = \lim_{h\to 0} \frac{cos(\pi+h)+1}{h}$$

or

$$f(x) = cos\ (\pi+x) \text{ and } a = 0 \quad \Longrightarrow \quad f'(0) = \lim_{h\to 0} \frac{cos(\pi+h)+1}{h}$$

Worksheet for Section 7

1. Make a careful sketch of $f(x) = sin\ x$ and below it sketch $f'(x)$ thinking slopes. What is $f'(x)$?

   answers may vary slightly here but the sketch should look like $f'(x) = cos\ x$

2. Using the **DEFINITION** of the derivative, find $g'(x)$ if $g(x) = \frac{1}{x^2}$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{h(x+h)^2 x^2} = \lim_{h \to 0} \frac{-2x - h}{(x+h)^2 x^2} = \frac{-2x}{x^4} = -2x^{-3}$$