MATH 150 HW Problems Key Exam 3

Homework Key for Section 1

1. Differentiate the following functions:

   (a) $f'(x) = 0$
   (b) $f'(x) = -\frac{2}{3}$
   (c) $f'(x) = 3x^2 - 5$
   (d) $f'(x) = \frac{5}{3}x^4$
   (e) $f'(x) = -\frac{2}{5}x^{-7/5}$
   (f) $f'(x) = 4\pi x^2$
   (g) $f'(x) = \frac{72}{x^7}$
   (h) $f'(x) = \frac{1}{2\sqrt{x}} - 2e^x$
   (i) $f'(x) = 2ax + b$
   (j) $f'(x) = 3/2x^{1/2}2x^{-1/2} - x^{-3/2}$
   (k) $f'(x) = e^{x+1}$

2. Find an equation of the tangent line to $f(x) = 3x^2 - x^3$ at the point$(1, 2)$.

$$y = 3x - 1$$

3. Find the first and second derivative of the following function:

$$f'(x) = 4x^3 - 9x^2 + 8 \quad f''(x) = 12x^2 - 18x$$

4. For what values of x does the graph $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent?

$$x \;=\; -1 \;\pm\; \frac{\sqrt{6}}{3}$$

Homework Key for Section 2

1. Differentiate:

   (a)
   $$f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$$

   (b)
   $$g'(x) = \frac{1}{2\sqrt{x}}e^x(2x + 1)$$

   (c)
   $$y' = \frac{e^x(x-2)}{x^3}$$

   (d)
   $$y' = \frac{xe^x}{(1+x)^2}$$

   (e)
   $$g'(x) = \frac{1}{(2x+1)^2}$$

   (f)
   $$f'(t) = \frac{8-2t^2}{(4+t^2)^2}$$

   (g)
   $$y' = 14x^6 - 4x^3 - 6$$

   (h)
   $$y' = \frac{-t-1}{(t-1)^3}$$

2. Use the Product Rule twice to prove that if f, g *and* h are differentiable then $(fgh)' = f'gh + fg'h + fgh'$

Homework Key for Section 3

1. Differentiate:

   (a) $f'(x) = 6x + 2\ sin\ x$

   (b) $f'(x) = cos\ x - \frac{1}{2}csc^2\ x$

   (c) $g'(x) = 3x^2cos\ x - x^3sin\ x$

   (d) $f'(\theta) = -csc\ \theta\ cot\ \theta + e^\theta(cot\ \theta - csc^2\ \theta$

   (e)
   $$y' = \frac{2 - tan\ x + x\ sec^2\ x}{(2 - tan\ x)^2}$$

   (f)
   $$y' = \frac{sec\ \theta\ tan\ \theta}{(1 + sec\ \theta)^2}$$

   (g)
   $$y' = \frac{x\ cos\ x - 2\ sin\ x}{x^3}$$

2. Find an equation of the tangent line to $y = x + cos\ x$ at the point$(0, 1)$

   $$y = x + 1$$

3. Find the following limit:

   $$\lim_{x\to 0} \frac{sin\ 3x}{x} = 3$$

Homework Key for Section 4

1. Differentiate:

   (a) $y' = -20x(1-x^2)^9$

   (b) $y' = e^{\sqrt{x}}/2\sqrt{x}$

   (c) $f'(x) = (20x^3+35)(x^4+7x-4)^4$

   (d) $f'(x) = (2+3x^2)/4(1+2x+x^3)^{3/4}$

   (e) $f'(x) = e^{-kx}(-kx+1)$

   (f) $y' = -24(2x^4+7)^5(8x-4)^{-4} + 40x^3(8x-4)^{-3}(2x^4+7)^4$

   (g) $y' = (cos\ x - x\ sin\ x)e^{xcos\ x}$

   (h) $y' = 2\ cos(tan\ 2x)sec^2(2x)$

   (i) $y' = 2^{sin\ \pi x}(\pi\ ln\ 2)cos\ \pi x$

   (j) $y' = (-\pi\ cos(tan\ \pi x)sec^2(\pi x)sin\sqrt{sin(tan\ \pi x)})/(2\sqrt{sin(tam\ \pi x)})$

   (k)
   $$y' = \frac{12x(x^2-1)^2}{(x^2+1)^4}$$

   (l)
   $$y' = \frac{1}{(x-1)^{1/2}(x+1)^{3/2}}$$

   (m)
   $$y' = (x^2+1)^{-3/2}$$

Homework Key for Section 5

1. Find dy/dx by implicit differentiation:

   (a)
   $$y' = -x^2/y^2$$

   (b)
   $$y' = \frac{2x + y}{2y - x}$$

   (c)
   $$y' = \frac{3y^2 - 5x^4 - 4x^3y}{x^4 + 3y^2 - 6xy}$$

   (d)
   $$y' = \frac{-2xy^2 - sin\ y}{2x^2y + x\ cos\ y}$$

   (e)
   $$y' = \frac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$$

   (f)
   $$y' = \frac{e^y\ sin\ x + y\ cos(xy)}{e^y\ cos\ x - x\ cos(xy)}$$

2. Find the derivative and simplify:

   (a)
   $$y' = \frac{(tan^{-1}\ x)^{-1/2}}{2(1 + x^2)}$$

   (b)
   $$y' = \frac{1 - x(1 + x^2)^{-1/2}}{1 + (x - \sqrt{1 + x^2})^2}$$