

Worksheet for Section 1

1. Find the derivatives of the following functions:

(a)

$$f(x) = \sqrt{30}$$

$$f'(x) = 0$$

(b)

$$f(x) = 5e^x + 3$$

$$f'(x) = 5e^x$$

(c)

$$f(x) = \frac{\sqrt{10}}{x^7}$$

$$f'(x) = -7\sqrt{10}x^{-8}$$

(d)

$$f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

2. Find the equation of the tangent line to $y = x^4 + 2e^x$ at $(0, 2)$

$$f'(x) = 4x^3 + 2e^x \implies \text{the slope at } (0, 2) \text{ is } f'(0) = 2$$

$$\text{so the equation is } y - 2 = 2(x - 0) \text{ or } y = 2x + 2$$

Worksheet for Section 2

1. Find the derivatives of the following functions:

(a)

$$f(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$$

$$f'(x) = \frac{x}{\sqrt{x}} - \frac{3x\sqrt{x}}{\sqrt{x}} = x^{1/2} - 3x$$

$$f'(x) = \frac{1}{2}x^{-1/2} - 3$$

(b)

$$g(x) = \sqrt{x}e^x$$

$$g'(x) = \sqrt{x} \left[\frac{d}{dx}(e^x) \right] + e^x \left[\frac{d}{dx}(\sqrt{x}) \right]$$

$$= \sqrt{x}e^x + e^x \frac{1}{2}x^{-1/2}$$

(c)

$$y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$$

$$y' = \frac{(\sqrt{x} + 1) \frac{d}{dx}(\sqrt{x} - 1) - [(\sqrt{x} - 1) \frac{d}{dx}(\sqrt{x} + 1)]}{(\sqrt{x} + 1)^2} = \frac{(\sqrt{x} + 1) \frac{1}{2}x^{-1/2} - [(\sqrt{x} - 1) \frac{1}{2}x^{-1/2}]}{(\sqrt{x} + 1)^2}$$

$$= \frac{x^{-1/2}}{(\sqrt{x} + 1)^2}$$

2. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$ find $(fg)'(3)$

$$(fg)'(3) = f(3)g'(3) + g(3)f'(3) = 4(5) + 2(-6) = 8$$

Worksheet for Section 3

1. Find the derivative of $f(x) = x \sin x$

$$f'(x) = x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x) = x \cos x + \sin x$$

2. Find an equation of the tangent line to $y = e^x \cos x$ at $(0, 1)$

$$y' = e^x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(e^x) = -e^x \sin x + e^x \cos x = e^x(\cos x - \sin x)$$

$$\text{at the point } (0, 1) \implies y' = e^0(\cos 0 - \sin 0) = 1 \implies y - 1 = x \text{ or } y = x + 1$$

3. Evaluate

$$\begin{aligned} & \lim_{t \rightarrow 0} \frac{\sin^2 3t}{t^2} \\ &= \lim_{t \rightarrow 0} \left(\frac{\sin 3t}{t} \right) \left(\frac{\sin 3t}{t} \right) = \left(\lim_{t \rightarrow 0} \frac{3 \sin 3t}{3t} \right) \left(\lim_{t \rightarrow 0} \frac{3 \sin 3t}{3t} \right) \\ &= 3^2 \left(\lim_{t \rightarrow 0} \frac{\sin 3t}{3t} \right)^2 = 3^2 = 9 \end{aligned}$$

Worksheet for Section 4

1. Find the derivative of the following functions:

(a) $F(x) = \sqrt{1 + 2x + x^3}$

$$F'(x) = \frac{1}{2}(1 + 2x + x^3)^{-1/2}(2 + 3x^2)$$

(b) $g(t) = \frac{1}{(t^4+1)^3}$

$$g'(t) = -3(t^4 + 1)^{-4}(4t^3)$$

(c) $y = \cos(x^3)$

$$y' = -\sin(x^3)(3x^2)$$

(d) $y = e^{-\pi x}$

$$y' = e^{-\pi x}(-\pi)$$

(e) $y = xe^{-x^2}$

$$y' = x \frac{d}{dx}(e^{-x^2}) + e^{-x^2} \frac{d}{dx}(x) = xe^{-x^2}(-2x) + e^{-x^2} = e^{-x^2}(1 - 2x^2)$$

(f) $y = e^{x \cos x}$

$$y' = e^{x \cos x} \frac{d}{dx}(x \cos x) = e^{x \cos x}(x(-\sin x) + \cos x)$$

(g) $F(z) = \sqrt{\frac{z-1}{z+1}}$

$$\begin{aligned} F'(z) &= \frac{1}{2} \left(\frac{z-1}{z+1} \right)^{-1/2} \frac{d}{dz} \left(\frac{z-1}{z+1} \right) = \frac{1}{2} \left(\frac{z-1}{z+1} \right)^{-1/2} \left(\frac{(z+1)(1) - (z-1)(1)}{(z+1)^2} \right) \\ &= \frac{1}{2} \left(\frac{z-1}{z+1} \right)^{-1/2} \left(\frac{z}{(z+1)^2} \right) \end{aligned}$$

Worksheet for Section 5

1. Find dy/dx by implicit differentiation:

(a) $\sqrt{x+y} = 1 + x^2y^2$

$$\frac{1}{2}(x+y)^{-1/2} \left(1 + \frac{dy}{dx} \right) = x^2 2y \frac{dy}{dx} + y^2 2x \iff \frac{1}{2\sqrt{x+y}} + \frac{dy/dx}{2\sqrt{x+y}} = 2x^2y \frac{dy}{dx} + 2xy^2$$

$$\begin{aligned} \frac{dy}{dx} \left[\frac{1}{2\sqrt{x+y}} - 2x^2y \right] &= 2xy^2 - \frac{1}{2\sqrt{x+y}} \iff \frac{dy}{dx} = \frac{2xy^2 - 1/(2\sqrt{x+y})}{1/(2\sqrt{x+y}) - 2x^2y} \\ &= \frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}} \end{aligned}$$

(b) $y^5 + x^2y^3 = 1 + ye^{x^2}$

$$5y^4y' + x^2 3y^2y' + y^3 2x = ye^{x^2} 2x + e^{x^2} y' \iff y'(5y^4 + 3x^2y^2 - e^{x^2}) = 2xye^{x^2} - 2xy^3$$

$$\implies y' = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}$$

2. Differentiate $y = \cos^{-1}(e^{2x})$

$$y' = -\frac{1}{\sqrt{1 - (e^{2x})^2}} \left(\frac{d}{dx}(e^{2x}) \right) = -\frac{2e^{2x}}{\sqrt{1 - e^{4x}}}$$