MATH 151 Final Review 1 Spring

1. Using a Maclaurin series you know evaluate $\int x^2 e^{-x^2}\ dx$.

2. Find the interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$

3. Determine if $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{e^n}$ is **AC**, **CC** or **D**.

4. Determine convergence or divergence of the following **<u>improper</u>** integral $\displaystyle\int_{-1}^{8} \frac{dx}{\sqrt[3]{x}}$. If convergent, evaluate.

5. Given $x = t - 2\sin(t)$ and $y = 1 - 2\cos(t)$ find:

   (a) $\frac{dy}{dx}$

   (b) $\frac{d^2y}{dx^2}$

6. Find the slope of the tangent line to $r = e^{3\theta}$ at $\theta = \frac{\pi}{2}$

7. Evaluate $\int \tan(x)\sec^5(x)\, dx$

8. Find the area bounded by one loop of $r = 4\cos(2\theta)$

9. Determine if the following **<u>improper</u>** integral converges or diverges: $\displaystyle\int_{0}^{\infty} \frac{dx}{1+x^2}$. If it converges, evaluate.

10. Using a Maclaurin series you know, find the Maclaurin series for $f(x) = x^2\sin(4x)$

11. Find the Taylor polynomial, $T_3(x)$ for $f(x) = \ln(x)$ centered at $a = 1$

12. Evaluate $\int e^{3x}\sin(2x)\, dx$

13. Evaluate $\int x\tan^{-1}(x)\, dx$

14. Evaluate $\int \sin^4(x)\cos^3(x)\, dx$

ANSWERS

1) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+3)(n!)}$

2) $\left[-\frac{1}{3}, \frac{5}{3}\right)$

3) Divergent, Ratio Test.

4) Converges, $\frac{9}{2}$

5a) $\frac{2\sin(t)}{1-2\cos(t)}$

5b) $\frac{2\cos(t)-4}{(1-2\cos(t))^3}$

6) -3

7) $\frac{\sec^5(x)}{5} + C$

8) 2π

9) Converges, $\frac{\pi}{2}$

10) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{2n+3}}{(2n+1)!}$

11) $(x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!}$

12) $\frac{4}{13}\left[-\frac{1}{2}e^{3x}\cos(2x) + \frac{3}{4}e^{3x}\sin(2x)\right] + C$

13) $\frac{x^2\tan^{-1}(x)}{2} - \frac{x}{2} + \frac{\tan^{-1}(x)}{2} + C$

14) $\frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7} + C$