1 Max and Min Values

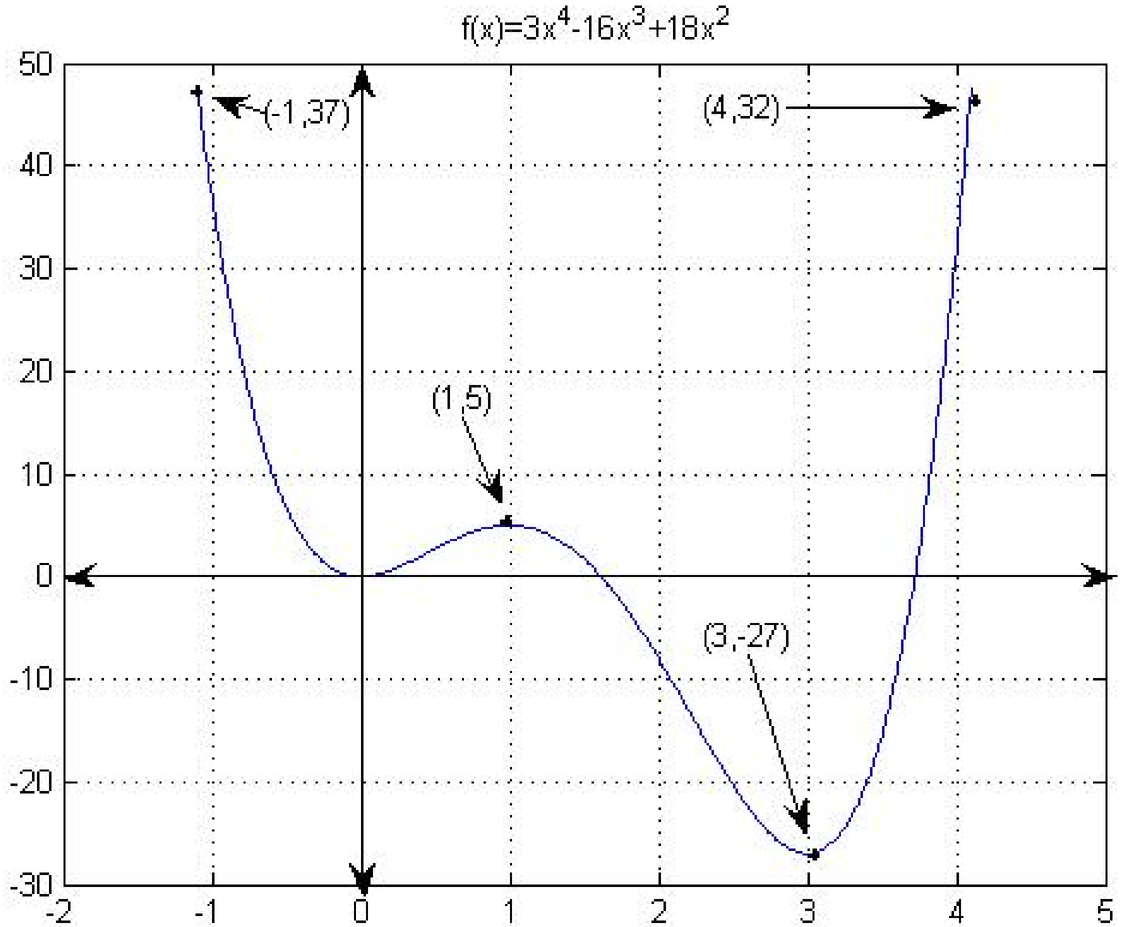
Using calculus to determine maximum and minimum values is much better than graphing the functions. Using the derivative you can be assured that you don't miss anything because of your graphing window

We will talk about two different kinds, *local* and *absolute or global*

A local max or min simply means that you have a max or min in some open interval

A global or absolute max or min means for all x, that is, the highest and lowest values for the entire graph

ex 1 Let's find any local and absolute max/min for the following



So we have

A local max at $x = 1$

An absolute max at $x = -1$, which is not local since its an endpoint

A local min at $x = 0$

A local and absolute min at $x = 3$

Neither at $x = 4$

Extreme Value Theorem

If f is a continuous function on a closed interval $[a, b]$, then f attains an absolute max, $f(c)$, and an absolute min, $f(d)$, at some numbers c and d in $[a, b]$

You need BOTH continuous and closed. I will show you in class why.

Now that we have defined these things the better question is how do we find them?

Fermat's Theorem

If f has a local min or max at c, and $f'(c)$ exists, then $f'(c) = 0$

BE CAREFUL

Just because $f'(0) = 0$ that does **NOT** imply that we have a local max or min. The classic counterexample is $y = x^3$

Also, you can have a local max or min even if $f'(c)$ DNE. Can you think of an example?

Definition

A *critical number* of a function f is a number c **in the domain of** f such that either $f'(c) = 0$ or $f'(c)$ DNE

ex 2 Find the critical numbers of $f(x) = 2x^3 + 3x^2 - 36x$

$f'(x) = 6x^2+6x-36 = 0 \iff x^2+x-6 = 0 \iff (x+3)(x-2) = 0 \implies$

$x = 2$ and $x = -3$ are critical numbers

So if f has a local max/min at c then c is a critical number of f

To find absolute max/min of a continuous function on a closed interval you need to:

1. Check the critical numbers and
2. Check the endpoints

ex 3 Find the absolute max and min of $f(x) = x^3 - 3x + 1$ on $[0, 3]$

First the critical numbers...

$$f'(x) = 3x^2 - 3 = 0 \iff x^2 - 1 = 0 \iff x = \pm 1$$

So we need to check how many values here?

$f(0)$, $f(1)$ and $f(3)$. Why not $f(-1)$?

$f(0) = 1$, $f(1) = -1$ and $f(3) = 19$

thus the absolute max is 19 and the absolute min is -1

Worksheet for Section 1

1. Sketch the graph of $f(x) = x^2 \;,\; 0 < x \leq 2$ and use your sketch to find the absolute and local max(s) and min(s).

2. Find the critical numbers of $g(x) = x^3 + x^2 + x$

Homework for Section 1

1. Sketch the graphs and find any absolute max/mins as well as local max/mins .

   (a) $f(x) = x^2 \ , \quad 0 < x < 2$

   (b) $f(x) = x^2 \ , \quad 0 \leq x < 2$

   (c) $f(x) = x^2 \ , \quad 0 \leq x \leq 2$

2. Find the critical numbers of the function:

   (a) $f(x) = 5x^2 + 4x$

   (b) $f(x) = x^3 + 3x^2 - 24x$

   (c) $f(x) = 3x^4 + 4x^3 - 6x^2$

3. Find the absolute mins and absolute maxs of f on the given intervals

   (a) $f(x) = 2x^3 - 3x^2 - 12x + 1 \ , \quad [-2, 3]$

   (b) $f(x) = x\sqrt{4 - x^2} \ , \quad [-1, 2]$

2 Mean Value Theorem

In order to prove the MEAN VALUE THEOREM we will need ROLLE'S THEOREM, so ...

ROLLE'S THEOREM

If a function f satisfies the following:

1. f is continuous on $[a, b]$
2. f is differentiable on (a, b)
3. $f(a) = f(b)$

$$\implies \exists\, c \in (a, b) \;\; \textit{such that} \;\; f'(c) = 0$$

Proof: (there are three cases)

1. If $f(x) = k$, where k is a constant

   Since $f'(x) = 0$, c can be any number in (a, b)
2. $f(x) > f(a)$ for some $x \in (a, b)$

   By the Extreme Value Theorem f has a max on $[a, b]$. Since $f(a) = f(b)$ it must attain this max in (a, b). Then f has a local max at c and since f is differentiable there, $f'(c) = 0$ by Fermat's Theorem.
3. $f(x) < f(a)$ for some $x \in (a, b)$

   Similar to case 2 except with a local min.

As a physics application, if an object is in the same position at two different times, $t = a$ and $t = b$, then $f(a) = f(b)$. This implies that there must exist a time between a and b where the velocity is zero.

Mean Value Theorem

If a function f satisfies the following:

1. f is continuous on $[a, b]$
2. f is differentiable on (a, b)

$$\Longrightarrow \quad \exists\, c \in (a, b) \;\; \textit{such that} \;\; f'(c) = \frac{f(b) - f(a)}{b - a}$$

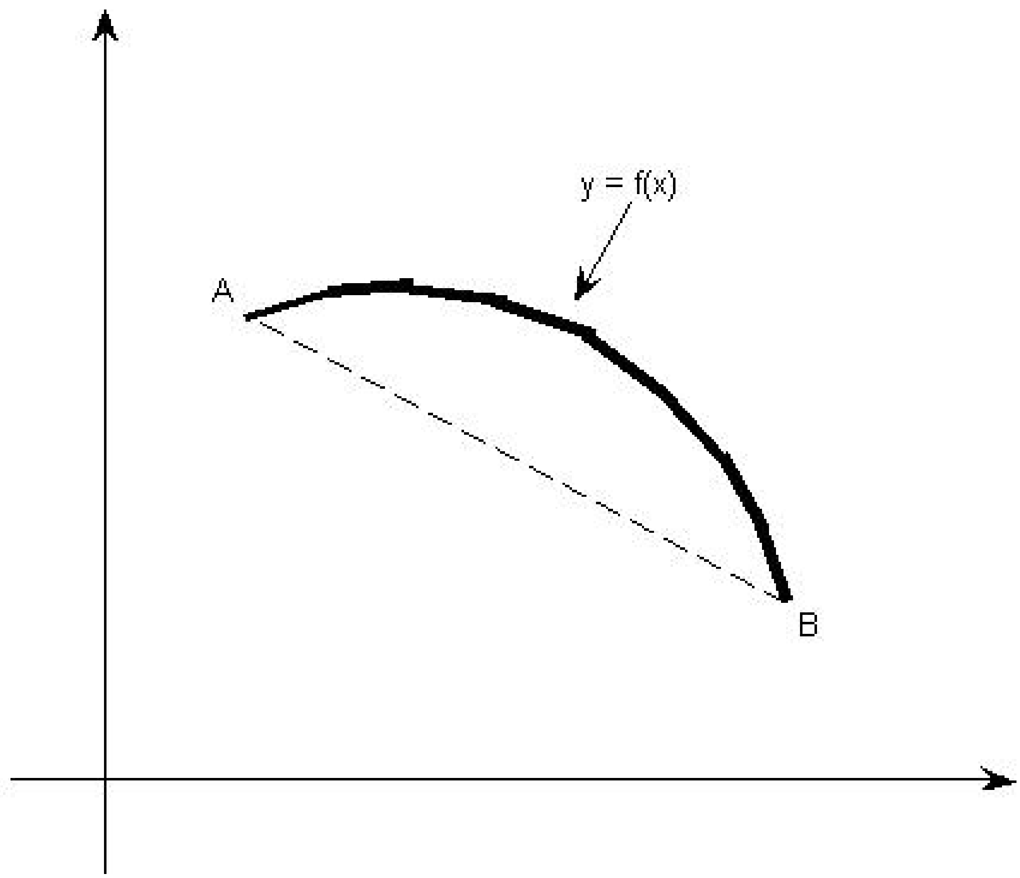
or

$$f(b) - f(a) = f'(c)(b - a)$$

What does this mean? (no pun intended)

I will illustrate with some functions in class.

Proof:



We shall define a new function h as the difference between f and the secant line AB and apply Rolle's Theorem to h

The equation of the secant line is

$$y = f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

so

$$h(x) = f(x) - f(a) - \frac{f(b) - f(a)}{b - a}(x - a)$$

Does h satisfy Rolle's ?

1. Is h continuous? Sure

2. Is h differentiable? Sure

$$h'(x) = f'(x) - \frac{f(b) - f(a)}{b - a}$$

3. $h(a) = h(b) = 0$

$$\begin{aligned} Thus \implies \exists\, c \text{ such that } 0 &= h'(c) = f'(c) - \frac{f(b) - f(a)}{b - a} \\ \implies f'(c) &= \frac{f(b) - f(a)}{b - a} \qquad \blacksquare \end{aligned}$$

The Mean Value Theorem says that there must exist a number where the instantaneous rate of change must equal the average rate of change over an interval.

ex 4 Suppose that $f(0) = -3$ and $f'(x) \le 5\ \ \forall\, x$. How large can $f(2)$ be?

Apply the MVT to $[0, 2]$. This implies that there must exist a c such that

$$f(2) - f(0) = f'(c)(2 - 0) \text{ so } f(2) = -3 + 2f'(c) \ \ since \ \ f'(x) \le 5 \implies 2f'(c) \le 10$$

therefore

$$f(2) = -3 + 2f'(c) \le -3 + 10 = 7$$

The largest $f(2)$ can be is 7

Theorem

$$If\ f'(x) \ = \ 0 \ \forall \, x \ \in \ (a, b) \ \implies \ f \ \ is\ constant\ on\ (a, b)$$

To prove this just pick any two values, x_1 and x_2, such that $x_1 < x_2$ and apply the MVT to $[x_1, x_2]$

Worksheet for Section 2

1. Verify that $f(x) = x^3 + x - 1$ on the interval $[0, 2]$ satisfies the hypothesis of the MVT. Then find all numbers c that satisfy the conclusion of the MVT.

2. Suppose that $3 \leq f'(x) \leq 5$ for all values of x. Show that $18 \leq f(8) - f(2) \leq 30$.

Homework for Section 2

1. Verify that $f(x) = 5 - 12x + 3x^2$ satisfies Rolle's Theorem and find any c that satisfies the conclusion.

2. Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no c in $(-1, 1)$ such that $f'(c) = 0$. Why is this not a counterexample?

3. Verify that $f(x) = 3x^2 + 2x + 5$ satisfies the MVT for $[-1, 1]$ and find any c that satisfies the conclusion.

4. Show that $f(x) = 1 + 2x + x^3 + 4x^5 = 0$ has exactly one real root.

5. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

6. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all values of x?

3 Derivatives and Graphs

If $f'(x) > 0$ on an interval I, then f is *increasing* on I

Proof:
Let $x_1, x_2 \in I$ with $x_1 < x_2$. By the MVT, $f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$
Since $f'(c) > 0 \implies x_2 - x_1 > 0 \implies f(x_2) - f(x_1) > 0 \implies f(x_1) < f(x_2)$

If $f'(x) < 0$ on an interval I, then f is *decreasing* on I

The proof for this one is similar to above

This information gives us the FIRST DERIVATIVE TEST

Suppose c is a critical number of a continuous function f:

1. If $f'(x)$ changes from positive to negative at c, then c is a local max.
2. If $f'(x)$ changes from negative to positive at c, then c is a local min.
3. If there is no sign change, then c is neither

ex 5 If $f(x) = 6x^4 + 16x^3 - 36x^2 - 14$, find where f is increasing, decreasing and any local max/mins

All of this is linked to the function's critical numbers so

$$f'(x) = 24x^3 + 48x^2 - 72x = 0 \iff x^3 + 2x^2 - 3x = 0$$
$$\iff x(x+3)(x-1) = 0$$

Thus the critical numbers are $x = 0, -3$ *and* 1 which gives us the following:

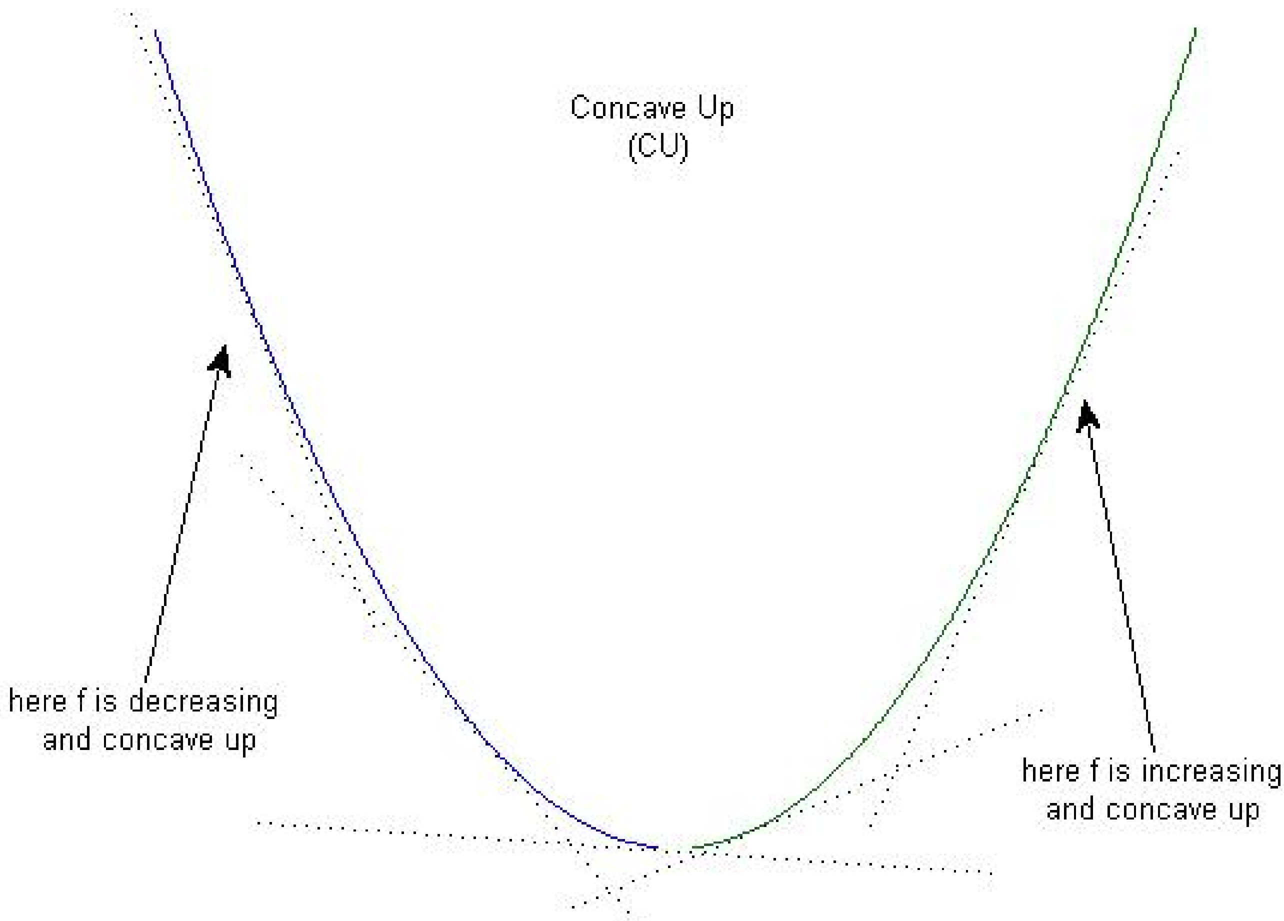
	$(-\infty, -3)$	$(-3, 0)$	$(0, 1)$	$(1, \infty)$
f'	-	$+$	-	$+$

Thus f is increasing on $(-3, 0)$ and $(1, \infty)$ and decreasing on $(-\infty, -3)$ and $(0, 1)$

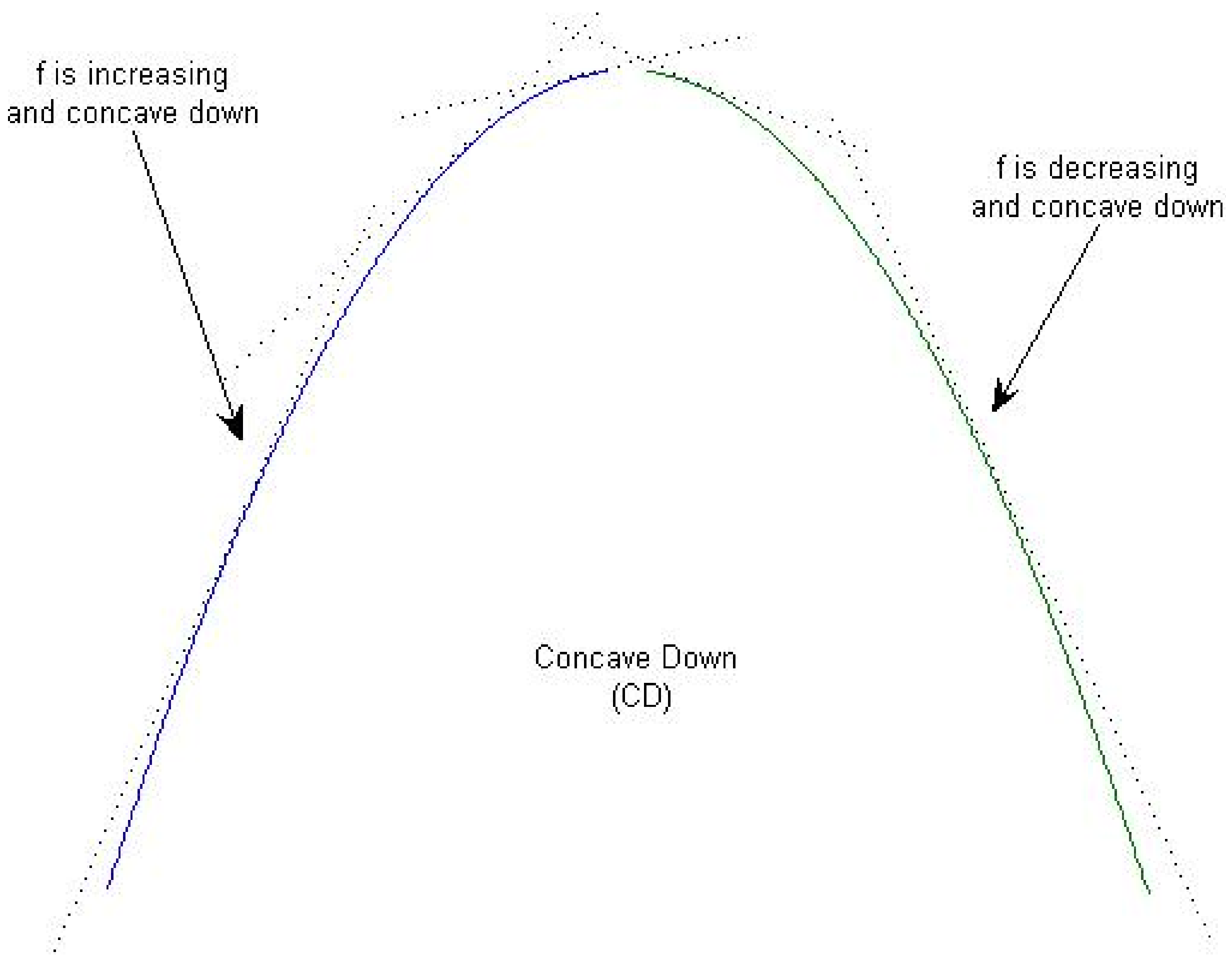
Also $x = -3$ and $x = 1$ are local mins and $x = 0$ is a local max

Definition

If a graph lies above all of its tangent lines on an interval I, then the graph is *concave up* or CU on I.



If a graph lies below all of its tangent lines on an interval I, then the graph is *concave down* or CD on I.



This gives us the TEST FOR CONCAVITY

1. If $f''(x) > 0$ on I, then f is CU on I.

   The reason being that $f''(x) > 0 \implies f' \; is \; increasing \implies CU$

2. If $f''(x) < 0$ on I, then f is CD on I.

   The reasoning here is similar.

Definition

A point P on the curve of $y = f(x)$ is an *inflection point* if f is continuous there and the curve changes concavity there.

which gives us the ...

Second Derivative Test

Suppose f'' is continuous near c, then:

1. If $f'(c) = 0$ *and* $f''(c) > 0 \quad \Longrightarrow \quad$ local min at c
2. If $f'(c) = 0$ *and* $f''(c) < 0 \quad \Longrightarrow \quad$ local max at c

ex 6 If $f(x) = x^4 - 4x^3$, find any local mins/maxs and points of inflection.

$$f'(x) \;=\; 4x^3 - 12x^2 \;=\; 0 \iff x^3 - 3x^2 \;=\; 0 \iff x^2(x-3) \;=\; 0$$

$\implies \quad x = 0$ and $x = 3$ are critical numbers

$$f''(x) \;=\; 3x^2 - 6x \;=\; 3x(x-2) \;=\; 0 \iff x = 0 \;\; \mathit{or} \;\; x = 2$$

$\implies \quad x = 0$ and $x = 2$ are *possible* inflection points

So, $f'(0) = 0$ and $f''(0) = 0$ so we need the first derivative test for $x = 0$. $f'(-1) < 0$ and $f'(1) < 0$ so $x = 0$ in neither a local max or min

$f'(3) = 0$ and $f''(3) > 0 \implies x = 3$ is a local min.

Also $f''(-1) > 0$ and $f''(1) < 0 \implies x = 0$ is an inflection point.

$f''(1) < 0$ and $f''(3) > 0 \implies x = 2$ is also an inflection point.

Worksheet for Section 3

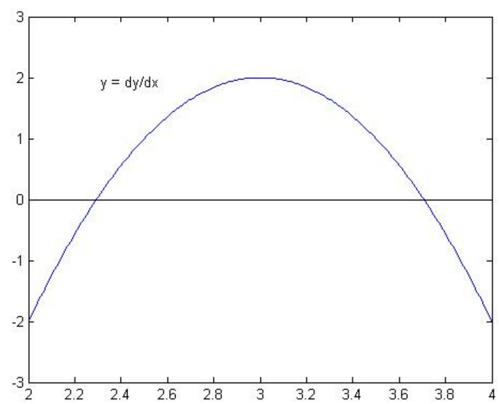
1. Find the intervals the function increases and decreases.
2. Find the local maximum and minimum values.
3. Find the intervals of concavity and the inflection points.

   (a) $h(t) = 3t^5 - 5t^3 + 3$

   (b) $g(x) = (x^2 - 1)^3$

Homework for Section 3

1. The graph of f' is below. On what what intervals is f increasing? Decreasing? What values of x are a local max? min?



2. If $f(x) = x^4 - 2x^2 + 3$
   (a) Find the intervals where f is increasing and decreasing
   (b) Find any local max and min
   (c) Find the intervals of concavity and inflection points.
3. Find any local max/min of $f(x) = x^5 - 5x + 3$ **using the 2nd derivative test**
4. If $f(x) = 2x^3 - 3x^2 - 12x$
   (a) Find the intervals where f is increasing and decreasing
   (b) Find any local max and min
   (c) Find the intervals of concavity and inflection points.
5. Sketch a graph that satisfies the following conditions:

$$f'(0) = f'(2) = f'(4) = 0$$

$$f'(x) > 0 \;\; if \;\; x < 0 \;\; or \;\; 2 < x < 4$$

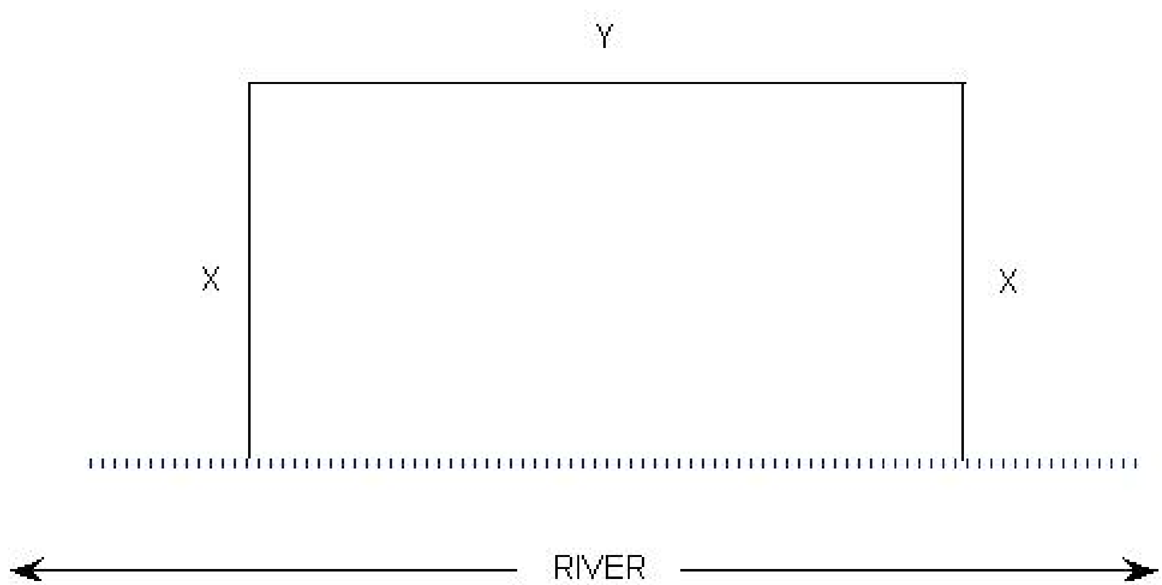
$$f'(x) < 0 \;\; if \;\; x > 4 \;\; or \;\; 0 < x < 2$$

$$f''(x) > 0 \;\; if \; 1 < x < 3$$
$$f''(x) < 0 \;\; if \; x < 1 \;\; or \;\; x > 3$$

4 Optimization

These differ from related rates in that you will only want an equation in one variable.

ex 7 Farmer Joe has 2400 feet of fence. He wants to fence off a rectangular region bordering a river. No fence is required along the river. What are the dimensions of the field with the largest area?



So $2x + y = 2400$ and he wants to maximize area which is $A = xy$

Since we need an equation in ONE variable

$$A = xy = x(2400-2x) = 2400x-2x^2 \implies A' = 2400-4x \;\; and$$
$$2400-4x = 0 \iff x = 600 \implies y = 1200$$

Thus the dimensions are 600 by 1200

ex 8 A cylindrical can is made to hold 1000 cm^3 of soda. Find the dimensions that will minimize the cost of the material to manufacture the can.

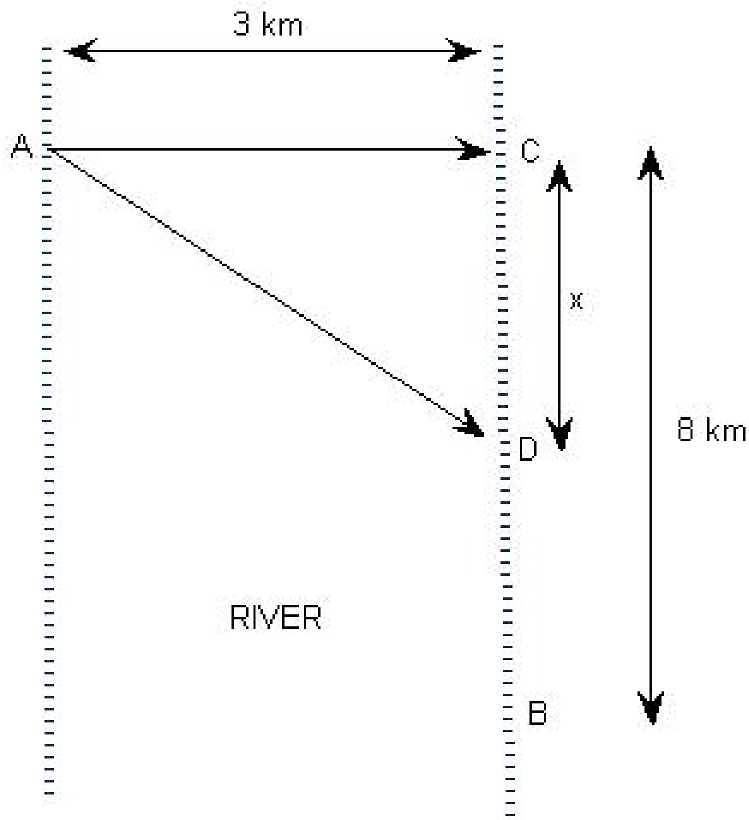
This is similar to asking you to minimize what? Surface area. So

$$SA = 2\pi r^2 + 2\pi rh \quad and \quad V = \pi r^2 h = 1000 \implies h = \frac{1000}{\pi r^2}$$

Thus

$$SA = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right) = 2\pi r^2 + \frac{2000}{r}$$
$$(SA)' = 4\pi r - \frac{2000}{r^2} = 0 \iff \frac{2000}{r^2} = 4\pi r \iff$$
$$\frac{2000}{4\pi} = r^3 \iff r = \sqrt[3]{\frac{500}{\pi}} \;\; and \;\; h = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2}$$

ex 9 Using the following diagram, the idea is to get from A to B as quickly as possible. You can row 6 km/hr and you can run 8 km/hr. Where should you land for the fastest time?



The time is represented by $\frac{distance}{rate}$ so a function that models the time it takes to get from A to B can be given by

$$T(x) = \frac{\sqrt{x^2+9}}{6} + \frac{8-x}{8}$$

Where AD represents the row and DB represents the run

So to minimize the time, let's minimize the function $T(x)$

$$T'(x) = \frac{x}{6\sqrt{x^2+9}} - \frac{1}{8} \quad \textit{and} \ T'(x) = 0 \iff \frac{1}{8} = \frac{x}{6\sqrt{x^2+9}}$$
$$\iff 6\sqrt{x^2+9} = 8x \iff \sqrt{x^2+9} = (4/3)x \iff x^2+9 = (16/9)x^2$$
$$\iff 9 = (7/9)x^2 \iff x^2 = \frac{81}{7} \quad so \ x = \sqrt{\frac{81}{7}}$$

Thus you should land at D where D is $\frac{9}{\sqrt{7}}$ below C for the fastest time.

Worksheet for Section 4

1. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

2. Show that of all the rectangles with a given area, the one with the smallest perimeter is a square.

3. Show that of all the rectangles with a given perimeter, the one with the greatest area is a square.

4. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.

5. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.

6. A cylindrical can without a top is made to contain a volume V of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

7. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is:

   (a) a maximum?

   (b) a minimum?

8. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

Homework for Section 4

1. Find two numbers whose difference is 100 and whose product is a minimum.

2. Find two positive numbers whose product is 100 and whose sum is a minimum.

3. Find the dimensions of a rectangle with perimeter 100 whose area is as large as possible.

4. Farmer Joe wants to fence 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this with a minimum cost for the fence?

5. A rectangular storage container is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides and top costs \$6 per square meter. Find the cost of the materials for the cheapest container.

6. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible surface area of such a cylinder.

5 Newton's Method

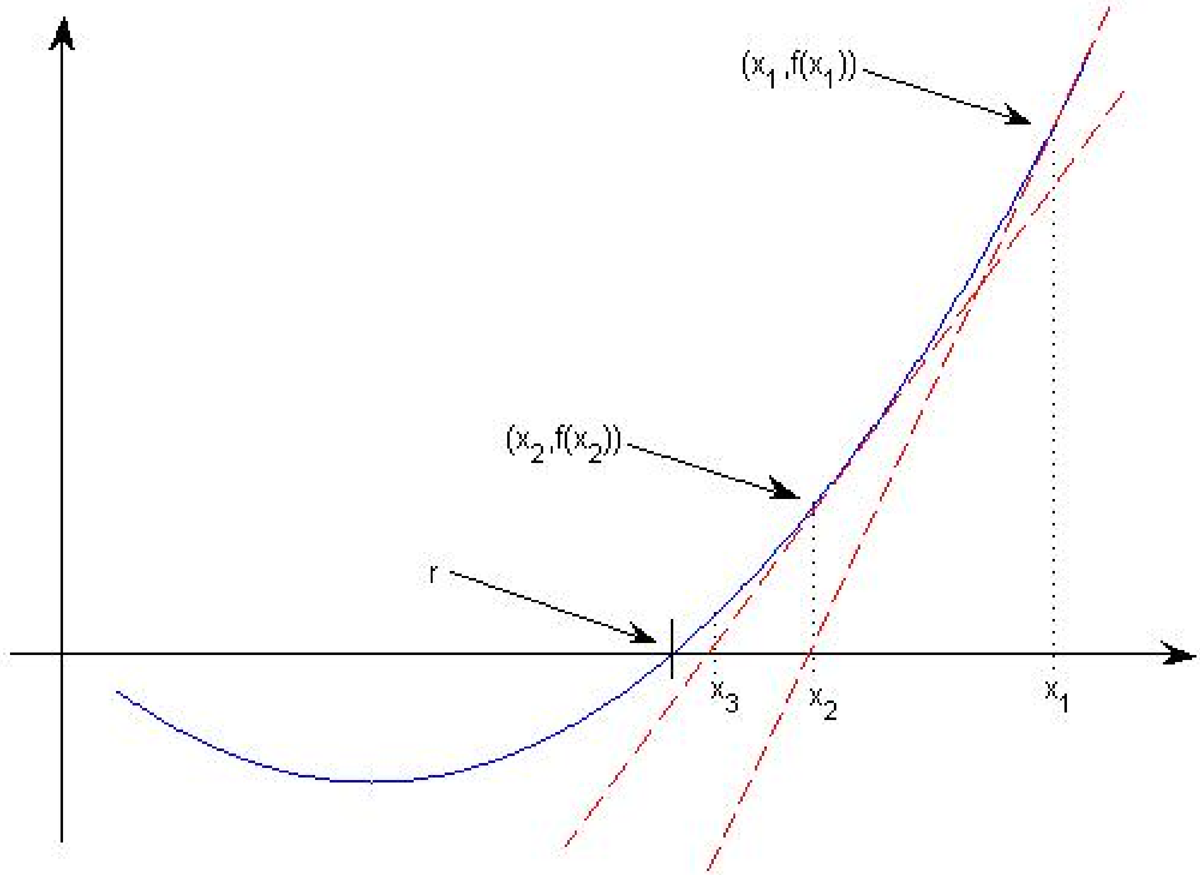
How do we find the roots of $ax^2 + bx + c$?

cubics?

Degree 4, 5 or higher?

How does your calculator find it?

Although your calculator can do this for you, it is important that you understand what is going on behind the scenes. It will also help to sharpen problem solving skills and critical thinking.



We start with an approximation, x_1, and consider the tangent line to the function at x_1. What is the tangent line's x-intercept? x_2

The idea is that since the tangent line is close to the curve, the tangent line's x-intercept will be close to the x-intercept of f, which is r.

What is the equation of the tangent line?

$$y - f(x_1) \;=\; f'(x_1)(x - x_1)$$

The x-intercept of the tangent line is when $y = 0$, thus

$$0 - f(x_1) = f'(x_1)(x_2 - x_1) \quad and\ if \quad f'(x_1) \neq 0 \implies$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

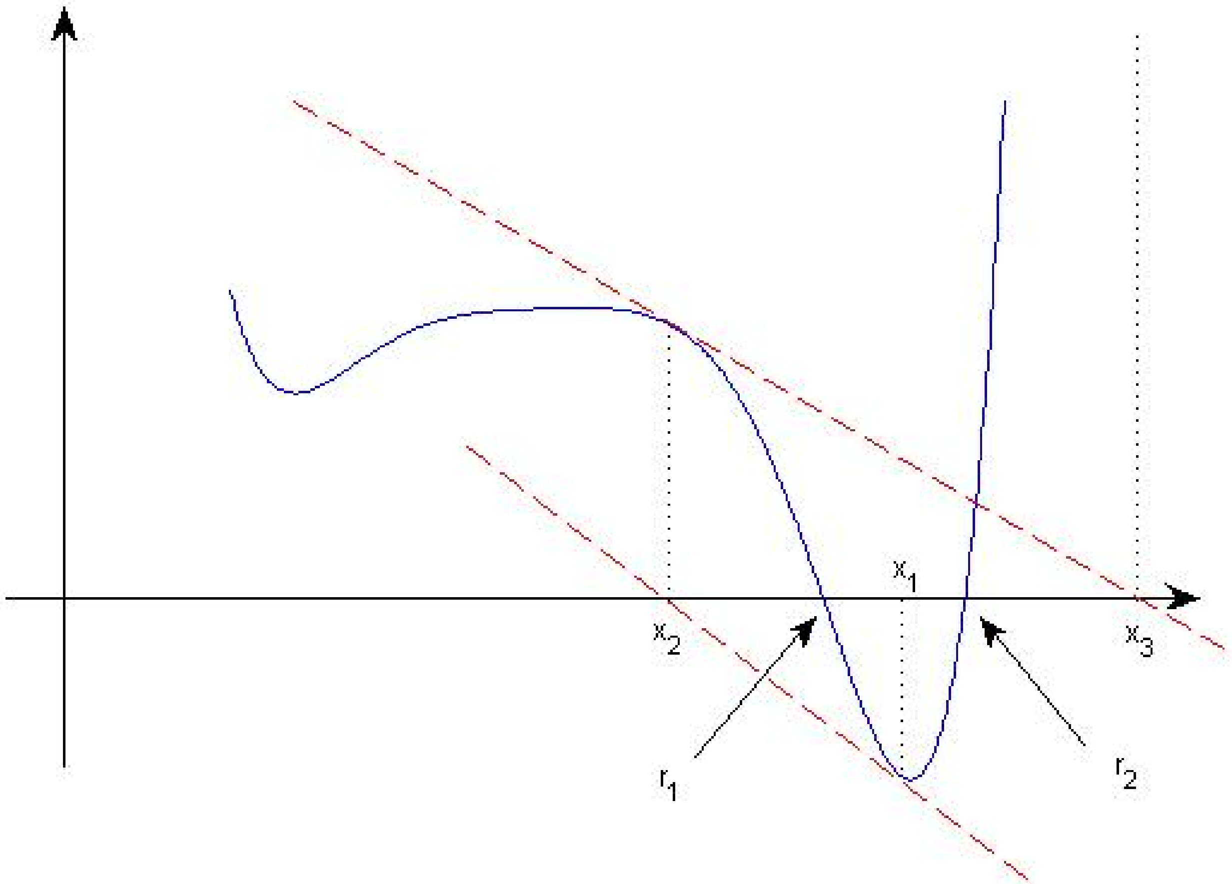
What about x_3?

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

In general, if $f'(x_n) \neq 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Does this method always work? It depends



Here x_3 is worse than x_1. Why?

ex 10 Use Newton's Method to approximate $\sqrt[3]{30}$ to 6 decimal places

We need a function with a root to use Newton's Method, so ...

$$x = \sqrt[3]{30} \iff x^3 = 30 \iff x^3 - 30 = 0$$

So

$$f(x) = x^3 - 30 \quad and \quad f'(x) = 3x^2 \implies$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \implies x_{n+1} = x_n - \frac{(x_n)^3 - 30}{3(x_n)^2}$$

Let $x_1 = 2.9$, then

$$x_2 = x_1 - \frac{(x_1)^3 - 30}{3(x_1)^2} = 2.9 - \frac{(2.9)^3 - 30}{3(2.9)^2} \approx 3.1111111$$

$$x_3 = x_2 - \frac{(x_2)^3 - 30}{3(x_2)^2} = 3.1111111 - \frac{(3.1111111)^3 - 30}{3(3.1111111)^2} \approx 3.1072373$$

$$x_4 \approx 3.10723250$$

$$x_5 \approx 3.10723251$$

Since the first 6 places agree, I can stop. Thus

$$\sqrt[3]{30} \approx 3.10723251$$

Worksheet for Section 5

1. If $x^3 - x^2 - 1 = 0$ and $x_1 = 1$, use Newton's method to find x_3.
2. Use Newton's method to approximate $\sqrt[3]{30}$ correct to eight decimal places.

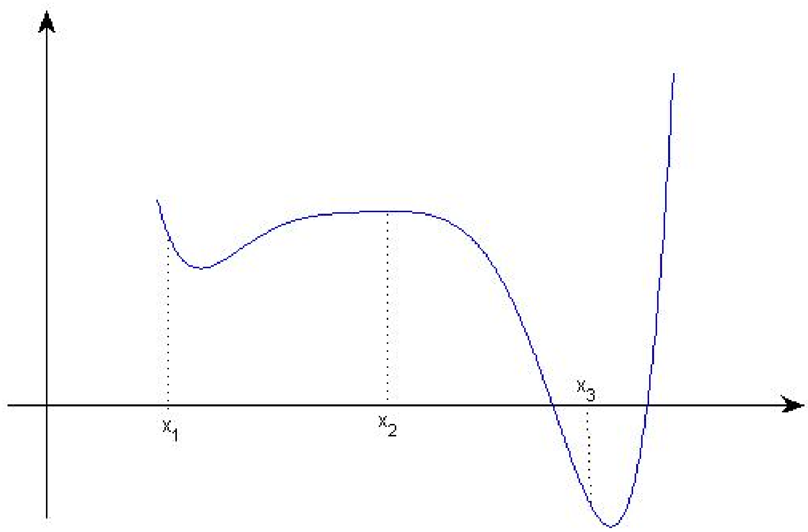
Homework for Section 5

1. Determine graphically what happens if Newton's method is used for each initial approximation.

   (a) x_1

   (b) x_2

   (c) x_3



2. Find x_3 if $x^3 + 2x - 4 = 0$ and $x_1 = 1$

3. Approximate the root of $x^4 - 2x^3 + 5x^2 - 6 = 0$ to six decimal places in the interval $[1, 2]$

4. Why would Newton's method not work to find the root of $x^3 - 3x + 6 = 0$ if $x_1 = 1$

6 Antiderivatives

Sometimes you know the velocity and want a particle's position.

Or you may know the rate of decay and want the mass at some time.

In fact, usually we observe rate of change and want to determine a function that models it.

So, how do we find a function, $F(x)$, whose derivative, $f(x)$, you know?

Definition

A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I

ex 11 If $f(x) = x^2$, can you guess what $F(x)$ might be?

Find a function $F(x)$ such that the derivative of it will give you x^2

How about $\frac{1}{3}x^3$? Any others work?

$$\frac{1}{3}x^3 + 16 \quad or \quad \frac{1}{3}x^3 + \pi \quad or \quad \ldots$$

Theorem

If F is an antiderivative of f on I, the most general antiderivative is:

$$F(x) + C \qquad where\ C\ is\ a\ constant$$

ex 12

1. Find the most general antiderivative of the following:
   (a) $f(x) = sin\ x$
   (b) $f(x) = x^n$
   (c) $f(x) = \frac{1}{x}$

YOU SHOULD BE FAMILIAR WITH THE FOLLOWING:

$f(x)$	General Antiderivative
$x^n \quad n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$ln \ \mid x \mid + C$
e^x	$e^x + C$
$sin\ x$	$-cos\ x + C$
$cos\ x$	$sin\ x + C$
$sec^2\ x$	$tan\ x + C$
$sec\ x\ tan\ x$	$sec\ x + C$
$\frac{1}{1+x^2}$	$tan^{-1}\ x + C$
$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1}\ x + C$

ex 13 Find f if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$ and $f(1) = 1$

So

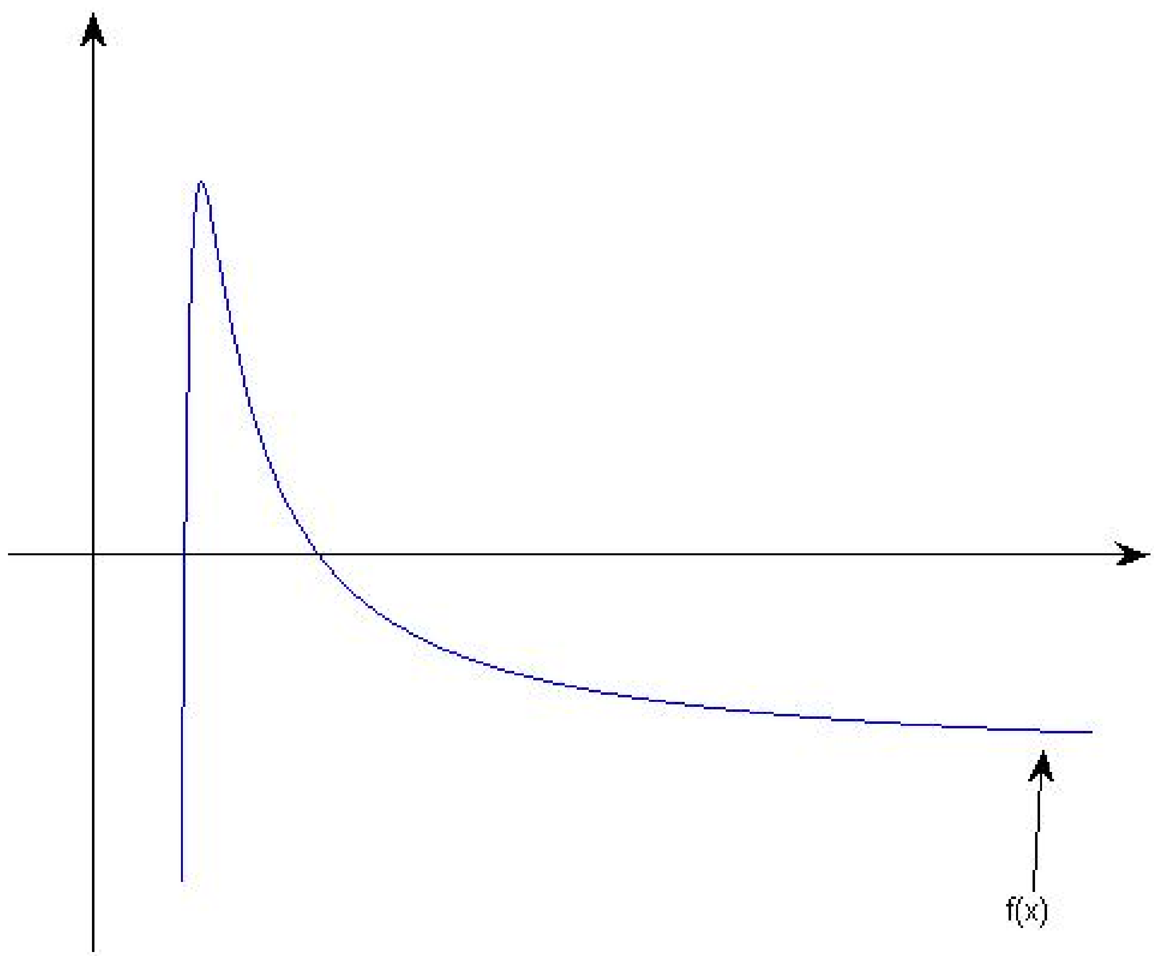
$$f''(x) = 12x^2 + 6x - 4 \implies f'(x) = 4x^3 + 3x^2 - 4x + C \implies$$
$$f(x) = x^4 + x^3 - 2x^2 + Cx + D$$

$$f(0) = 4 \implies 4 = D \;\; and \;\; f(1) = 1 \implies$$
$$1 = 1^4 + 1^3 - 2(1)^2 + C + 4 \implies C = -3$$

Thus

$$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

ex 14 Given the graph of $f(x)$, graph $F(x)$ on the same axes given that $F(start\ point) = 1$



ex 15 If a particle has acceleration, $a(t) = 10 + 3t - 3t^2$, with $s(0) = 0$ and $s(2) = 10$, find the position function.

$$a(t) = 10 + 3t - 3t^2 \implies v(t) = 10t + (3/2)t^2 - t^3 + C \;\; and$$
$$s(t) = 5t^2 + (1/2)t^3 - (1/4)t^4 + Ct + D$$
$$Since \;\; s(0) = 0 \implies D = 0 \;\; and \;\; s(2) = 10 \implies$$
$$10 = 5(2)^2 + (1/2)(2)^3 - (1/4)(2)^4 + 2C \implies C = -5$$

Thus

$$s(t) = 5t^2 + (1/2)t^3 - (1/4)t^4 - 5t$$

Worksheet for Section 6

1. Find f if $f''(x) = 2 + x^3 + x^6$.
2. Find f if $f''(x) = 2 - 12x$, $f(0) = 9$ and $f(2) = 15$.
3. A particle is moving with velocity $v(t) = sin\ t - cos\ t$. Find its position if $s(0) = 0$.

Homework for Section 6

1. Find the most general antiderivative of the following:

   (a) $f(x) = x - 3$

   (b) $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$

   (c) $f(x) = (x+1)(2x-1)$

   (d) $f(x) = \frac{10}{x^9}$

   (e) $f(x) = \frac{x^4+3\sqrt{x}}{x^2}$

2. Find f for the following:

   (a) $f''(x) = 6x + 12x^2$

   (b) $f''(x) = (2/3)x^{2/3}$

   (c) $f'''(x) = e^x$

   (d) $f'(x) = 2\ cos\ x + sec^2\ x, \quad -\pi/2 < x < \pi/2, \quad f(\pi/3) = 4$

   (e) $f'(x) = x^{-1/3}, \quad f(1) = 1, \quad f(-1) = -1$

   (f) $f''(x) = 24x^2 + 2x + 10, \quad f(1) = 5, \quad f'(-1) = 3$

3. If a particle's velocity is $v(t) = sin\ t - cos\ t$, find its position if $s(0) = 0$.