MATH 254 Homework Part 2

Homework for Section 1

1. Use coordinate vectors to determine the linear independence of the following set of polynomials.

$$1 - 2t^2 - 3t^3\ ,\ t + t^3\ ,\ 1 + 3t - 2t^2$$

Homework for Section 2

1. Determine the dimensions of $Nul\ A$ and $Col\ A$ for:

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -1 \\ 0 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. Let B be the basis of $\mathbb{P}_2$ consisting of the first three Laguerre polynomials which are $1, 1-t, 2-4t+t^2$. Let $p(t) = 7 - 8t + 3t^2$. Find the coordinate vector of p relative to B.

Homework for Section 3

1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   Find

   (a) rank A

   (b) dim Nul A

   (c) a basis for $Col\ A$

   (d) a basis for $Row\ A$

   (e) a basis for $Nul\ A$

2. If A is a 4x3 matrix, what is the largest possible dimension of the row space of A? If A is a 3x4 matrix, what is the largest possible dimension of the row space of A? Explain.

Homework for Section 4

1. Let

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

   Find a basis for the eigenspace corresponding to $\lambda = 4$

2. Construct an example of a 2x2 matrix with only one distinct eigenvalue.

Homework for Section 5

none

Homework for Section 6

none

Homework for Section 7

none

Homework for Section 8

none

Homework for Section 9

1. Suppose a vector y is orthogonal to vectors u and v. Show that y is orthogonal to the vector $u + v$

Homework for Section 10

1. Is the following set of vectors orthonormal? If it is only orthogonal, then normalize them.

$$\begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \quad \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix}$$

Homework for Section 11

none

Homework for Section 12

1. The following set of vectors is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

$$\begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix}$$

Homework for Section 13

1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$

   Find a least squares solution of $Ax = b$ by

   (a) constructing the normal equations for $\hat{x}$ and

   (b) solving for $\hat{x}$

Homework for Section 14

none

Homework for Section 15

1. Let

$$\mathbf{A} = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix}, \quad \mathbf{v_1} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Verify that v_1 and v_2 are eigenvectors of A, then orthogonally diagonalize A.