MATH 151 Exam 6 Review Key Spring

1. $\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

$$x = 3\ sec\ \theta \quad , \quad dx = 3\ sec\ \theta\ tan\ \theta\ d\theta$$

$$\frac{1}{6}sec^{-1}\left(\frac{x}{3}\right) - \frac{\sqrt{x^2-9}}{2x^2} + C$$

2. $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}\,dx$ $[Hint: \text{Let } u = \sqrt[6]{x}]$

$$x = u^6 \quad , \quad \sqrt{x} = u^3 \quad , \quad dx = 6u^5 \quad , \quad \sqrt[3]{x} = u^2$$

$$2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ ln\left|\sqrt[6]{x} - 1\right| + C$$

3. Find the Maclaurin series for $f(x) = sin^2\ x$.

note that you must do this one the long way, you can't use $sin\ x$

$$\frac{2}{2!}x^2 - \frac{2^3}{4!}x^4 + \frac{2^5}{6!}x^6 - \frac{2^7}{8!}x^8 + \ldots$$

4. $\int x^3\sqrt{x^2+4}\,dx$

$$2^5\left[\frac{sec^5\ \theta}{5} - \frac{sec^3\ \theta}{3}\right] + C \ = \ 2^5\left[\frac{(\frac{\sqrt{x^2+4}}{x})^5}{5} - \frac{(\frac{\sqrt{x^2+4}}{x})^3}{3}\right] + C$$

5. $\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$

$$ln\mid x \mid + \frac{1}{2}ln(x^2+4) - \frac{1}{2}tan^{-1}\left(\frac{x}{2}\right) + C$$

6. $\int_0^3 \frac{1}{x-1}\,dx$

this is an improper integral, undefined at 1

$$= \lim_{t\to 1^-} \int_0^t \frac{1}{x-1}\,dx + \lim_{t\to 1^+} \int_t^3 \frac{1}{x-1}\,dx \ = \ \ldots \text{ diverges}$$

7. Find **R** and **I** for $\sum_{n=2}^{\infty} \frac{(-1)^n\ x^n}{4^n\ ln\ n}$

$$R = 4 \quad , \quad I = (-4, 4]$$

8. Evaluate as a power series: $\int \frac{ln(1-t)}{t}\,dt$

$$C - \sum \frac{t^n}{n^2}$$

9. If $f(x) = e^{x^2}$, find $T_5(x)$.

$$e^{x^2} \approx 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \frac{x^{10}}{5!}$$

10. $\int ln\ x\,dx$

integration by parts $\ldots$ $x\ ln\ x - x + C$

11. Find **I** for $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$

$$I = [2, 4)$$

12. Find the Maclaurin series for $f(x) = cos\ \sqrt{x}$

$$\sum (-1)^n \frac{x^n}{(2n)!} = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \ldots$$

13. $\int_0^1 x5^x\,dx$

integration by parts $\ldots$ $\frac{5}{ln\ 5} - \frac{4}{(ln\ 5)^2}$

14. Find **I** for $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

$$(-\infty, \infty)$$